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Henning T. Sogaard and Claus G. Sørensen

Model for Optimisation of Farm Machinery Sizes

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Summary

This report describes the theory and implementation of a general non-linear programming model for an overall optimisation of the sizes of the field machines at a given farm. The optimal machines are defined as those minimising the annual costs associated with machinery and labour. The machinery costs include both fixed costs and operating costs. The primary decision variables in the model are the sizes of the machines and the number of tractors and their size.

The optimisation model takes a number of limitations and constraints into account. The most important among these can be expressed as follows: the available number of man-hours, machine-hours and tractor-hours are limited; the number of tractors should be large enough to satisfy the operation which requires most tractors to be in operation simultaneously; each operation is only relevant within a certain range of weeks; the operations must be performed in proper succession; each type of machinery is only available in a certain range of sizes; the capacity of a set of machines depends on the size of the individual machines and whether the machines are working by turns or simultaneously.

In the model, two aspects, which are crucial in the optimisation of machinery size, are taken into consideration, namely timeliness and workability. The timeliness of the performance of an operation may affect the total costs significantly. To complete an operation in due time the machinery must be of reasonable size, but large machinery as well as delayed execution of an operation may be associated with significant costs. Therefore, the model seeks the optimal balance between timeliness and machinery size.

The workability of the crop and soil, which is mainly determined by the weather, affects the extent to which the labour and machinery can be utilised. If the workability coefficient associated with a given operation is low, then most of the working hours can not be used for this operation, due to unfavourable soil, crop and/or weather conditions. Consequently, such operations may call for large machinery – i.e. large capacity – to ensure a high rate of performance when the conditions are good.

An important goal when formulating the model has been to minimise its complexity. In order to fulfil this, it has been necessary to introduce a few approximations in some of the equations defining the model.

To quantify the parameters in the model, data defining the characteristics of the farm and its possibilities of mechanisation must be supplied. Most of these data are related to the field operations which have to be performed during one season (e.g. field areas, type of machinery to

be used, agronomic windows of operations, workability coefficients, timeliness coefficients, expected crop yields and cost coefficients). Another substantial part of the data characterises the possible machinery to be used (prices, operating costs, possible sizes etc.). Besides that, data specifying the available labour during the season must also be supplied.

For the implementation of the model, the software package GAMS (General Algebraic Modelling System) has been applied. When using GAMS it is possible to formulate mathematical programming models in a high level language which to a certain extent resembles the underlying mathematical formulation of the model.

The GAMS model consists of a main source file which defines the model in terms of parameters, decision variables and equations. On the basis of data which are read from separate data files, the parameters are quantified, and initial values of the decision variables are produced. When the non-linear solver has been run, the information describing the optimal solution will be reported in an output file.

The model has been tested by using it for optimisation of the machinery at a case farm. The farm is a dairy farm located in the western part of Denmark. The optimal machinery found for this farm has been compared with the existing machinery at the farm, and the accordance is good. However, a few significant differences have been found indicating that not all of the existing machines are of optimal size.

Sammendrag (summary in Danish)

Denne rapport beskriver teorien for og implementeringen af en generel ikke-lineær programmeringsmodel til samlet optimering af størrelserne af markmaskinerne på en given bedrift. De optimale maskinstørrelser er defineret som dem, der minimerer de årlige omkostninger til maskiner og arbejdskraft. I maskinomkostninger er inkluderet både faste og variable omkostninger. De primære beslutningsvariable i modellen er maskinstørrelserne samt antallet og størrelserne af traktorerne.

Optimeringsmodellen tager hensyn til en række bindinger og begrænsninger. De vigtigste blandt disse udtrykker følgende: De disponible mand-, maskin- og traktortimer er begrænsede; der skal være nok traktorer til den operation, der kræver flest traktorer samtidigt; udførelsen af en operation er kun relevant inden for en begrænset periode; operationerne skal udføres i rigtig rækkefølge; hver maskintype fås kun i et begrænset størrelsesinterval; kapaciteten af et sæt af maskiner afhænger af størrelserne af de enkelte maskiner, og hvorvidt maskinerne arbejder på skift eller samtidigt.

I modellen tages der hensyn til to forhold, som har afgørende betydning for optimeringen af maskinstørrelserne. Det drejer sig om rettidseffekt og andelen af mulige operationstimer. Rettidigheden for udførelsen af en operation kan have stor betydning for de samlede omkostninger. For at kunne færdiggøre en operation rettidigt kræves maskiner af en vis størrelse, men store maskiner såvel som forsinket udførelse af en operation kan være forbundet med betydelige omkostninger. Derfor søger modellen den optimale balance mellem rettidighed og maskinstørrelse.

Andelen af mulige operationstimer for en operation er den brøkdel af tiden, som operationen forventeligt kan udføres på, når der tages hensyn til, at jordens og/eller afgrødens tilstand samt vejrforholdene må forventes at forhindre operationens udførelse i visse perioder. Muligheden for at arbejde med jord og afgrøde har stor indflydelse på, hvor god udnyttelse man kan få af arbejdskraft og maskiner. Hvis andelen af mulige operationstimer for en given operation er lille, betyder det, at en stor del af arbejdstiden ikke kan bruges på denne operation på grund af ugunstige betingelser, hvad angår jord, afgrøde og/eller vejr. Derfor kan sådanne operationer stille krav om store maskiner og dermed stor kapacitet for at sikre hurtig gemmenførelse af operationen, når forholdene er gunstige.

Et vigtigt mål ved formuleringen af modellen har været at minimere kompleksiteten. For at opnå dette har det været nødvendigt at indføre nogle få approksimationer i de ligninger, der definerer modellen.

Til kvantificeringen af modellens parametre må der fremskaffes data, der karakteriserer den aktuelle bedrift samt dens muligheder, hvad angår mekanisering. De fleste af disse data knytter sig til de markoperationer, der skal udføres på bedriften i løbet af en sæson (f.eks. markstørrelser; hvilken slags maskiner, der skal bruges; relevante tidsperioder for operationernes udførelse; andelen af mulige operationstimer; koefficienter for rettidseffekt; forventet høstudbytte og omkostningskoefficienter). En anden betydelig del af data karakteriserer de maskiner, der er mulighed for at benytte (priser, variable omkostninger, mulige størrelser osv.). Desuden kræves også data vedrørende den tilgængelige arbejdskraft hen over sæsonen.

Modellen er implementeret ved hjælp af software-pakken GAMS (General Algebraic Modeling System). GAMS gør det muligt at formulere matematiske programmeringsmodeller i et højniveausprog, som i nogen grad minder om den bagved liggende matematiske formulering af modellen.

GAMS-modellen består af en kildefil, hvori modellen er defineret ved hjælp af parametre, beslutningsvariable og ligninger. Kvantificeringen af parametrene og beregningen af startværdier for beslutningsvariablene sker på basis af data, som indlæses fra særskilte datafiler. Efter kørsel af den ikke-lineære løsningsalgoritme gemmes relevant information om den fundne optimale løsning i en uddatafil.

Modellen er blevet testet ved at finde den optimale maskinpark for en case-bedrift. Det drejer sig om en kvægbedrift, beliggende i det vestlige Danmark. Ved sammenligning af den beregnede optimale maskinpark med den eksisterende maskinpark på bedriften blev der fundet god overensstemmelse. Der blev dog konstateret nogle få betydelige forskelle, som vidner om, at ikke alle maskiner på den pågældende bedrift er størrelsesmæssigt optimale.

1 Introduction

In recent years, the agricultural primary sector has experienced an increased focusing on the ability of farmers to make the available resources as productive as possible within market, environmental and regulatory constraints. Among the resources considered, labour and machinery overshadow all other cost categories, and much is to be gained by adapting and operating these factors efficiently within the boundaries of the actual needs arising from farm size, crop plans, etc. Studies conducted on Danish farms show vast differences in machinery costs ranging from 1500 to 6000 DKK per hectare (The National Committee for Buildings and Machinery, 1995¹). Such figures emphasise the relevance of developing methods for choosing the optimal machinery sizes.

This report describes a model for determination of the optimal technical capacity. The model is to be used as a decision support tool, both when analysing different farm machinery systems separately and as an integrated part of the overall farm simulation model FASSET, developed by the effort of a Danish multi-disciplinary research project (Jacobsen *et al.*, 1998). In the latter case the model is used for the specification of the initial conditions and expectations prior to a dynamic simulation of the farm development over a number of years (Rasmussen and Dalsgaard, 1994).

Generally, the identification of an optimal mechanisation level is a very complex process involving the interactions between machines and between the farm machinery system and biological and meteorological subsystems, such as crop, soil, weather, etc. The following questions all have to be answered systematically: which requirements concerning the working operation in question have to be met by the technical equipment? Which machines are available on the market? Which machinery sizes are economically optimal? Which types of costs are accumulated during use of the machinery? The model presented in this report specifically undertakes the task of sizing the machines and as a part of that estimating the costs, while the selection of the types of machinery must be done manually prior to the use of the model. If one or more operations are to be accomplished by contractors, machinery for those operations should not be included in the system of machinery.

Some of the earliest systems and models to support strategic decision making within the domain of farm machinery management were relatively simple, static and deterministic. Hunt (1983) included variables for quantification of the timeliness of operations. Other approaches have involved simulation (e.g. Audsley and Boyce, 1974), linear programming techniques (e.g. Nilsson, 1972; Cairol and Jannot, 1990), or a combination of these modelling and solu-

¹ All monetary values mentioned in this report are based on 1995 prices.

tion techniques (e.g. Kline *et al.*, 1989). Glen (1987) gives a comprehensive survey of different models proposed for determination of equipment requirements on a farm. One of the conclusions is that models in this domain often end up very complex requiring large quantities of not easily accessible input data.

The approach described in this report involves the development of an optimisation model based on a level of aggregation consistent with the accessible and existing data related to machinery sets, crops, weather, timeliness of operations, etc. The formulation of the model is kept at a minimum of complexity by limiting the number of constraints and variables. The main reason for this is that the optimisation model is intended to be integrated as part of an overall farm simulation model.

The objectives of the work presented in this report are as follows:

- to develop a general non-linear programming model for optimisation of the sizes of the farm machines based on a least-cost concept
- to implement the model by use of the programming software GAMS (General Algebraic Modelling System) (Brooke *et al.*, 1992) and
- to test and validate the model with a realistic data set.

2 Design, mathematical formulation and implementation of the optimisation model

The purpose of the non-linear optimisation model is to find the least-cost sizes of the machines² in the farm machinery system given data concerning the operations to be performed during the year. Before the formal description of the model is given in terms of mathematical equations³, the set of decision variables used in the model will be defined, and the units associated with various quantities in the model will be listed.

2.1 Units

Table 1 shows the system of units used for variables and constants in the model.

² The term “machine” is used for real machines as well as agricultural implements.

³ The term “equation” covers both real equations and inequalities.

Table 1. Units associated with various quantities

Quantity	Unit
Time consumption when performing an operation	Hours (h)
Time of the year	Weeks
Travelling distance, working width	Metres (m)
Weight	Metric tonnes (t)
Value (prices)	Danish kroner (DKK)
Tractor power	Watts (W)
Field area	Square metres (m ²)
Field speed of tractor/machinery	Metres per hour (m/h)

The reason why area is measured in square metres instead of hectares, and field speed is measured in metres per hour instead of kilometres per hour is that the distance measure used is metres. The consequence is that any use of conversion factors between units is avoided.

2.2 Decision variables and indices

The indices described below are the domains over which variables, parameters and systems of equations and inequalities are defined.

- i is used for numbering of machines in the farm machinery system ($i = 1, \dots, N^M$), where N^M is the total number of machines⁴ (exclusive tractors)
- j is used for numbering of the operations to be performed during the year ($j = 1, \dots, N^O$) where N^O is the total number of operations and
- k is used for numbering of the weeks during the year ($k = 1, \dots, 52$).

The decision variables defined in the model are as follows:

x_i^M is the size of the i 'th machine ($i = 1, \dots, N^M$). The size of a machine is either measured as theoretical working width (in metres, e.g. for harrows and ploughs), theoretical harvesting capacity (in tonnes per hour, e.g. for combines and exact choppers) or load capacity (in tonnes, e.g. for trailers).

⁴ In this report superscripts (e.g. M in N^M) are *not* used as exponents. A superscript should be considered a part of the variable name.

x^T is the power (in Watt) of the tractors in the farm machinery system. To simplify the model all tractors are assumed to be identical as to power.

z^T is the number of tractors in the farm machinery system.

x_j^O is the effective field capacity of the machine or set of machines⁵ used for the performance of the j 'th operation ($j = 1, \dots, N^O$). The measuring unit is either m^2/s or t/h , depending on the size unit(s) associated with the machine (or set of machines) used for the operation in question.

$x_{j,k}$ is the fraction of the j 'th operation being performed in the k 'th week ($0 \leq x_{j,k} \leq 1$; $j = 1, \dots, N^O$; $k = 1, \dots, 52$).

The primary decision variables in the model are the machinery sizes together with the number of tractors and their sizes. The effective field capacity of whole sets of machines (x_j^O , $j = 1, \dots, N^O$) is introduced to simplify the model formulation. The decision variables $x_{j,k}$ ($j = 1, \dots, N^O$; $k = 1, \dots, 52$) describe how the performance of the operations is distributed in time (with a time resolution of one week). The main purpose of introducing these variables is to enable modelling of timeliness and how it influences the total costs.

2.3 Mathematical formulation of the model

The set of equations and inequalities which define the optimisation model is listed below. The model is defined in terms of the decision variables and indices described above together with a number of parameters which characterise the specific optimisation problem. The decision variables are the unknowns which can be found by solving the optimisation model, while the parameters have to be provided before the solution can be found. Further explanation of parameters and separate parts of the model is given subsequently. In Appendix F the symbols and notations used in the model formulation are listed.

Optimisation criterion:

$$\min_{x_i^M, x_j^O, x_{j,k}, x^T, z^T} f(x_i^M, x_j^O, x_{j,k}, x^T, z^T) = \sum_i (\varphi_{1,i} x_i^M + \varphi_{0,i}) + \psi z^T x^T + \sum_k \sum_j x_{j,k} \left(\alpha_j + \frac{\beta_j + \gamma_j x^T}{x_j^O} + \delta_j |k - t_j^{\text{opt}}| \right) \quad (1)$$

Minimise the total annual costs

Constraints:

⁵ A set of machines used for the performance of a given operation will be referred to as a "machinery set" in the following sections.

- $$\sum_j \frac{r_j x_{j,k} A_j U_j}{x_j^O w_j} \leq T_k, \quad \forall k \quad (2) \quad \text{Available man-time is limited.}$$
- $$\sum_{j \in M_j} \frac{x_{j,k} A_j U_j}{x_j^O w_j} \leq T^W, \quad \forall k, i \quad (3) \quad \text{Available machinery-time is limited.}$$
- $$\sum_j \frac{q_j x_{j,k} A_j U_j}{x_j^O w_j} \leq z^T T^W, \quad \forall k \quad (4) \quad \text{Available tractor-time is limited.}$$
- $$z^T \geq q_j, \quad \forall j \quad (5) \quad \text{The number of tractors must satisfy the most tractor demanding operation.}$$
- $$x_{j,k} = 0, \quad \forall j, k | k < t_j^{\min} \vee k > t_j^{\max} \quad (6) \quad \text{Limited time period (number of weeks) for performance of each operation.}$$
- $$\sum_k x_{j,k} = 1, \quad \forall j \quad (7) \quad \text{All operations must be completed.}$$
- $$\sum_{\kappa=1}^k x_{i,\kappa} \geq \sum_{\kappa=1}^k x_{j,\kappa}, \quad \forall j, k, i | i \in F_j \quad (8) \quad \text{The operations must be performed in proper sequence.}$$
- $$x_i^{M,\min} \leq x_i^M \leq x_i^{M,\max}, \quad \forall i \quad (9) \quad \text{Lower and upper limits on the machine sizes.}$$
- $$\frac{1}{x_j^O} = \begin{cases} \max_i \frac{s_{jj}}{x_i^M}, & \text{if } h_j = 0 \\ \sum_i \frac{s_{jj}}{x_i^M}, & \text{if } h_j = 1 \end{cases}, \quad \forall j \quad (10) \quad \text{Relationship between the effective field capacity and the sizes of individual machines.}$$
- $$x^T \geq \theta_i x_i^M, \quad \forall i \quad (11) \quad \text{The power of the tractors must fit the most power demanding machine.}$$
- $$x_j^O \geq 0, \forall j; \quad x_{j,k} \geq 0, \forall j, k; \quad x^T \geq 0 \quad (12) \quad \text{The continuous decision variables must be non-negative.}$$
- $$z^T \in \{0, 1, 2, \dots\} \quad (13) \quad \text{The number of tractors must be a non-negative integer.}$$

Notice that the mathematical symbol \forall which should be read “for all” has been applied. Also notice that expressions of the form “ \forall indices | condition(s)” should be read “for all values of indices satisfying condition(s)”.

The model includes a non-linear cost function to be minimised in (1) and a number of linear and non-linear constraints in (2)-(13). The new parameters introduced through these equations are explained below.

2.4 Objective function

The structure of the objective function in (1) is based on theories published by Hunt (1983) and Have (1991) and includes fixed costs as well as operating costs calculated on an annual basis. The objective function is a sum of three terms. In the first term, $\sum_i (\varphi_{1,i} x_i^M + \varphi_{0,i})$, the fixed costs (interest, depreciation, etc.) associated with the machinery are calculated as a sum over all machines. In this expression it has been assumed that the purchase price, P_i^M , of machine i is a linear function of its size:

$$P_i^M = p_{1,i}^M x_i^M + p_{0,i}^M$$

It should be noted that Have (1991) assumes direct proportionality between size and price, but our studies show that a general linear relationship is more appropriate.

The fixed annual costs are assumed to be a given fraction, c_i^M , of the purchase price which means that the fixed annual costs of machine i can be calculated as

$$c_i^M P_i^M = c_i^M (p_{1,i}^M x_i^M + p_{0,i}^M) = \varphi_{1,i} x_i^M + \varphi_{0,i}$$

where $\varphi_{0,i} = c_i^M p_{0,i}^M$ and $\varphi_{1,i} = c_i^M p_{1,i}^M$.

In the second term of the objective function ($\psi^T x^T$) the fixed costs associated with tractors are calculated. Here, the purchase price, P^T , of a tractor is taken to be proportional to its size, x^T , (power): $P^T = p^T x^T$. Furthermore, the fixed annual costs of a tractor are assumed to be a given fraction, c^T , of the purchase price and can be computed as: $c^T P^T = c^T p^T x^T = \psi x^T$, where $\psi = c^T p^T$. Thus, the total fixed annual costs associated with z^T tractors of size x^T add up to $\psi z^T x^T$.

The third term in the objective function

$$\sum_k \sum_j x_{j,k} \left(\alpha_j + \frac{\beta_j + \gamma_j x^T}{x_j^O} + \delta_j |k - t_j^{\text{opt}}| \right) \quad (14)$$

is a sum of the variable machinery and timeliness costs over all weeks ($k = 1, \dots, 52$) and operations ($j = 1, \dots, N^0$). This expression is based on a cost function for a single machine. For an exact chopper, for instance, the following expression is used for calculation of the operating costs, C_v :

$$C_v = \frac{AU}{Ke} (r(p_{k1}K + p_{k0}) + B_k K + L + dP) \quad (15)$$

where

- A [m^2] is the area to be harvested,
- B_k [DKK/t] is the expenses for fuel and oil per operating hour and per unit of theoretical harvesting capacity,
- dP [DKK/h] is the expected repair and maintenance costs of the tractor per working hour, expressed as a product of a coefficient, d [DKK/(W h)], and the tractor size, P [W],
- e [$0 < e \leq 1$] is the field efficiency which expresses the relationship between gross and net capacity,
- K [t/h] is the theoretical harvesting capacity of the exact chopper,
- L [DKK/h] is the labour cost,
- p_{k0} [DKK] and
- p_{k1} [DKK/(t/h)] are parameters to be used when calculating the purchase price of an exact chopper as a linear function of the purchase price (price = $p_{k1}K + p_{k0}$),
- r [h^{-1}] is the expected repair and maintenance costs of the exact chopper per working hour expressed as a fraction of the purchase price,
- U [t/ m^2] is the crop yield.

Equation (15) is based on and is very similar to the cost function described by Have (1991).

By definition let

$$x^0 = Ke, \quad x^T = P, \quad \alpha = \frac{AU}{e} (rp_{k1} + B_k), \quad \beta = AU(p_{k0} + L), \quad \text{and} \quad \gamma = AUd,$$

where the effective field capacity, x^0 , and the tractor size, x^T , are decision variables. Now, (15) can be rewritten as follows:

$$C_v = \alpha + \frac{\beta + \gamma x^T}{x^0} \quad (16)$$

which can be recognised as a part of (14).

The formula in (16) is generally applicable to harvesting machinery, but it can also be used for the two other types of machinery, i.e. for machinery where size is defined either by theoretical working width or load capacity. However, the definitions of x^O , α , β and γ depend on the type of machinery. If the size of a given machine is defined by theoretical working width, the operating costs are calculated as

$$C_v = \frac{A}{vbe} (r(p_{b1}b + p_{b0}) + B_b b + L + dP) \quad (17)$$

Through this equation five new quantities have been introduced:

- b [m] is the theoretical working width,
 B_b [DKK/(m h)] is the expenses for fuel and oil per operating hour and per unit of theoretical working width,
 p_{b0} [DKK] and
 p_{b1} [DKK/m] are parameters to be used when calculating the purchase price of the machine as a linear function of the purchase price (price = $p_{b1}b + p_{b0}$), and
 v [m/h] is the driving speed in the field.

Now define

$$x^O = vbe, \quad x^T = P, \quad \alpha = \frac{A}{ve} (rp_{b1} + B_b), \quad \beta = A(rp_{b0} + L), \quad \text{and} \quad \gamma = Ad$$

The combination of these definitions with (17) will lead to the expression on the right-hand side of (16).

Now assume that the size of a given machine is defined by its load capacity (normal for trailers). In that case the operating costs can be calculated as

$$C_v = \frac{M\tau}{m} (r(p_{m1}m + p_{m0}) + B_m m + L + dP) \quad (18)$$

where

- τ [h] is the time used for transportation of one load,
 B_m [DKK/(t h)] is the expenses for fuel and oil per operating hour and per unit load capacity,

m [t] is the load capacity,
 M [t] is the mass to be transported,
 p_{m0} [DKK] and
 p_{m1} [DKK/t] are parameters to be used when calculating the purchase price of the machine as a linear function of the purchase price (price = $p_{m1}m + p_{m0}$).

The following definitions are now introduced:

$$x^O = \frac{m}{\tau}, \quad x^T = P, \quad \alpha = M\tau(rp_{m1} + B_m), \quad \beta = M(rp_{m0} + L), \quad \gamma = Md$$

The use of these definitions together with (18) will once again result in the expression in (16).

The considerations described above prove that the cost expression in (16) can be used for all machinery types. Furthermore, (16) can be applied for operations involving a number of machines working either simultaneously or by turns. This means that an individual set of parameters, α , β and γ , can be calculated for each operation based on corresponding parameters for each of the machines used in the particular operation. Appendix A explains how to do that. In the model parameter values specific to operations are applied, and this explains the j -subscripts in (14).

The timeliness costs of operation j are calculated in the expression $\delta_j |k - t_j^{\text{opt}}|$ in (14). The parameter t_j^{opt} denotes the optimum week for the performance of operation j (as far as optimisation of crop return is concerned). The loss of crop return is assumed to be proportional to the time interval between t_j^{opt} and the actual week, k , where the operation is performed (see Have, 1991). The proportionality constant is δ_j [DKK/week].

From the above explanations it can be seen that the operating costs and timeliness costs associated with the performance of operation j in week k add up to

$$\alpha_j + \frac{\beta_j + \gamma_j x_j^T}{x_j^O} + \delta_j |k - t_j^{\text{opt}}|$$

However, if only a certain fraction, $x_{j,k}$, of operation j is performed in week k , where $0 \leq x_{j,k} \leq 1$, the costs will be reduced to

$$x_{j,k} \left(\alpha_j + \frac{\beta_j + \gamma_j x_j^T}{x_j^O} + \delta_j |k - t_j^{\text{opt}}| \right)$$

while the remaining part of the costs will be “moved” to other weeks where the rest of the operation is performed. By this, the expression which appears after the sum signs in (14) has been derived.

2.5 Constraints

The optimisation model includes 12 groups of constraints in (2)-(13).

The inequalities (2)-(4) are introduced to ensure that the available number of man-hours, machinery-hours and tractor-hours are not exceeded. To better understand these inequalities it should be noticed that the expression

$$\frac{x_{j,k} A_j U_j}{x_j^o} \quad (19)$$

gives the time consumption (hours) associated with the fraction of operation j which is performed in week k (this expression corresponds exactly to the quantity $t_{j,k}$ defined in Appendix B). The parameter A_j denotes the field area to be treated, while the meaning of U_j depends on the unit used for x_j^o . If the unit of x_j^o is t/h, which is the case in operations where materials are applied to or removed from the field (e.g. application of slurry or grain harvest), U_j is simply the applied or removed amount per unit of area (t/m²). If the unit of x_j^o is m²/h, U_j is 1 (e.g. harrowing and ploughing).

The inequalities (2) state that the total number of man-hours used by the operations in a given week, k , should be less than T_k , which is the expected number of man-hours available for field work during week k . The total number of man-hours used during week k is computed by summing up the man-hour consumption for all operations. The man-hour consumption of a given operation, j , is computed by multiplying the duration of the operation (see (19)) by the number of workers, r_j , involved fulltime in the operation and dividing it by a workability factor, w_j . The workability factor ($0 \leq w_j \leq 1$) of an operation is defined as the fraction of the working hours which is left for the performance of the operation when the expected hours with unfavourable weather, soil or crop conditions have been left out. More details on this subject can be found in Appendix B.

The inequalities (3) express that an arbitrarily chosen machine, i , cannot run for more than T^w hours in any week, k . The parameter T^w is the number of working hours during one week. Notice that summation on the left hand side of the inequalities is only running over those operations, j , which make use of the machine, i . This is the reason for employing the notation

$j | i \in M_j$, where M_j denotes the set of machines used in operation j . The inequalities (2) and (3) are based on the same theory, which is explained further in Appendix B.

The inequalities (4) ensure that the number of tractor-hours used in an arbitrarily chosen week, k , is less than $z^T T^w$ (the number of tractors at the farm multiplied by the number of working hours during one week). The left hand side of the inequalities is very similar to the left hand side of (2); the only difference is that r_j is replaced by q_j , which is the number of tractors used in operation j . For further information, see Appendix B.

The inequalities in (5) ensure that the number of tractors at the farm is large enough to satisfy the operation which requires most tractors to work simultaneously. Through the inequalities in (6) it is possible to restrict the performance of operation j to a limited number of weeks ($t_j^{\min}, t_j^{\min} + 1, \dots, t_j^{\max}$). It would, for instance, be natural to limit spring sowing to the spring weeks. The main purpose of (6) is to reduce the number of free decision variables, thereby reducing the complexity of the model without introducing a real reduction of the decision space of the optimisation problem. The constraints in (7) simply express that all operations should be 100% completed. The inequalities in (8) ensure that the operations are performed in correct succession. The set F_j introduced in these inequalities denotes the set of operations which must precede operation j . Thus, (8) expresses that if $i \in F_j$ (i.e. operation i is to precede operation j), then the completed fraction of operation j (e.g. sowing) must not exceed the completed fraction of operation i (e.g. harrowing) in any week (k).

A given machinery type is only available within a limited range of sizes. This is expressed through (9), where $x_i^{M,\min}$ and $x_i^{M,\max}$ are the minimum and maximum size of machine i , respectively. The equations in (10) give the relationships between the effective field capacity of the machinery sets used for the operations and the sizes of the individual machines. If the machines in the machinery set are operating simultaneously ($h_j = 0$), then the effective field capacity will be determined by the “slowest” machine, i.e. the machine which has the largest inverse effective capacity ($\max_i s_{j,i} / x_i^M$). If the machines in the machinery set are operating by turns ($h_j = 1$), then the inverse effective field capacity will be the sum of the inverse effective capacities of all the machines involved ($\sum_i s_{j,i} / x_i^M$). In the special case where only one machine is used, $h_j = 0$ and $h_j = 1$ will lead to the same result. The parameter $s_{j,i}$ is a proportionality constant which determines the relation between the size of machine i and its effective capacity in operation j (size = $s \times$ effective capacity). If machine i is not used in operation j , then $s_{j,i} = 0$ should be applied. More details on how to calculate $s_{j,i}$ can be found in Appendix A. The inequalities in (11) ensure that the tractor size is adapted to the machine which requires most tractor power. The parameter θ_i denotes the required tractor power per size unit of machine i .

The constraints in (12) and (13) are simple non-negativity conditions applying to continuous and discrete decision variables. Notice that the non-negativity requirement on x_i^M has already been satisfied through (9). Furthermore, notice that the requirement $0 \leq x_{j,k} \leq 1$ is satisfied through (7) together with (12) ($x_{j,k} \geq 0$).

2.6 Implementation of the model

The model described above has been implemented by use of the high-level programming software GAMS (General Algebraic Modelling System, Brooke *et al.*, 1992). Appendix C shows a print of the program code, and in Appendix D the structures of the files with input data are described.

The program code can be divided into three parts: (1) definition of the model, (2) solution of the model and (3) post-processing and saving of the results. The model definition section involves definition of sets, parameters, tables, variables and equations. The identifiers introduced in connection with these definitions have been chosen in such a way that the correspondence with the variable and parameter names in Section 2.3 in most cases can be seen directly (the parameter α_j in Section 2.3 e.g. corresponds to ALPHA (J) in the program).

Since the non-linear model is solved by using an iterative algorithm, initial values for the decision variables have to be specified. These initial values are generated automatically by the program in a way which most likely will produce a feasible model solution.

The equation definitions in the program are reproductions in the GAMS notation of the mathematical equations in Section 2.3, with the exception of the equations in (10) which have been reformulated to the following form before rewriting them in the GAMS notation:

$$\left\{ \begin{array}{ll} x_j^O \leq \frac{x_i^M}{S_{j,i}}, & \forall j, i | i \in \mathbf{M}_j \wedge h_j = 0 \\ x_j^O \leq \frac{1}{\sum_i \frac{S_{j,i}}{x_i^M}}, & \forall j | h_j = 1 \end{array} \right.$$

The non-linear programming algorithm CONOPT is used for solution of the model. The “GAMS/CONOPT User Notes” by Drud (1999) can be found at the internet address <http://www.gams.com/solvers/conopt/page1.htm>.

Appendix E contains an example of the output results produced by the GAMS model code. The data correspond to optimisation run 2, described in Chapter 3.

3 Test and validation – example

The case farm used for test and validation is located in the western part of Denmark and has a total acreage of 81 ha. The crop plan includes 5.3 ha of winter rye, 7.6 ha of winter barley, 9.2 ha of spring barley, 17.5 ha of fodder beets, 31.9 ha of whole crop for silage and 9.5 ha of grass. Fodder beets and grass/silage are grown for livestock feeding purposes, while the grain crops are grown to be sold from the farm. The livestock plan includes 80 dairy cows and 88 young cows. The following machinery resources are available on the farm: conventional plough (three-furrow, 1.05 m), mower (1.65 m), slurry tanker (6 t), seed bed cultivator (5 m), straw baler (7 t/h), sowing machine (4 m), flail forage harvester (1.5 m, 20 t/h), straw grating plant (2 m), roller (5 m), beet harvester (1.6 m, 26 t/h), sprayer (12 m), tipping trailer (4.5 t), precision seed drill (2.5 m), fertiliser applicator (16 m, 4.5 t) and universal trailers. There are no combines or exact choppers available on the farm, and operations where such machinery is required are performed by a contractor.

The main part of the input data for the model is either related to operations or machinery. For each operation different parameters should be quantified, e.g. field identification, area of the field, type of machinery involved, number of workers involved, agronomic window of operation, workable time, timeliness of operation, crop yield, cost coefficients, etc. Each machine considered should be characterised by a number of economic parameters, which include new value and fuel costs specifically related to either working width, harvesting capacity or load capacity. Other cost factors are repair and maintenance as a function of new value, timeliness costs estimated on a weekly basis, hourly wages and prices of produce harvested and sold from the farm. Furthermore, each machinery type should be characterised by minimum and maximum size and its need for tractor power as a function of its size. The description of the input data files presented in Appendix D gives a survey of the data needed to solve the model.

The farm specific data are all collected from the actual farm, while economic coefficients, data for timeliness effects, etc., are specified on the basis of different publications (among others Machinery Survey, 1994; Olsen, 1977; Pedersen, 1989; ASAE, 1990). Regression functions for the purchase price related to working width, harvesting capacity or load capacity have been identified, while other calculations are carried out to estimate economic coefficients concerning operating costs. The linear relationships between prices and sizes of machinery are shown in Figure 1, Figure 2 and Figure 3. Table 2 shows some of the economical figures behind the case study.

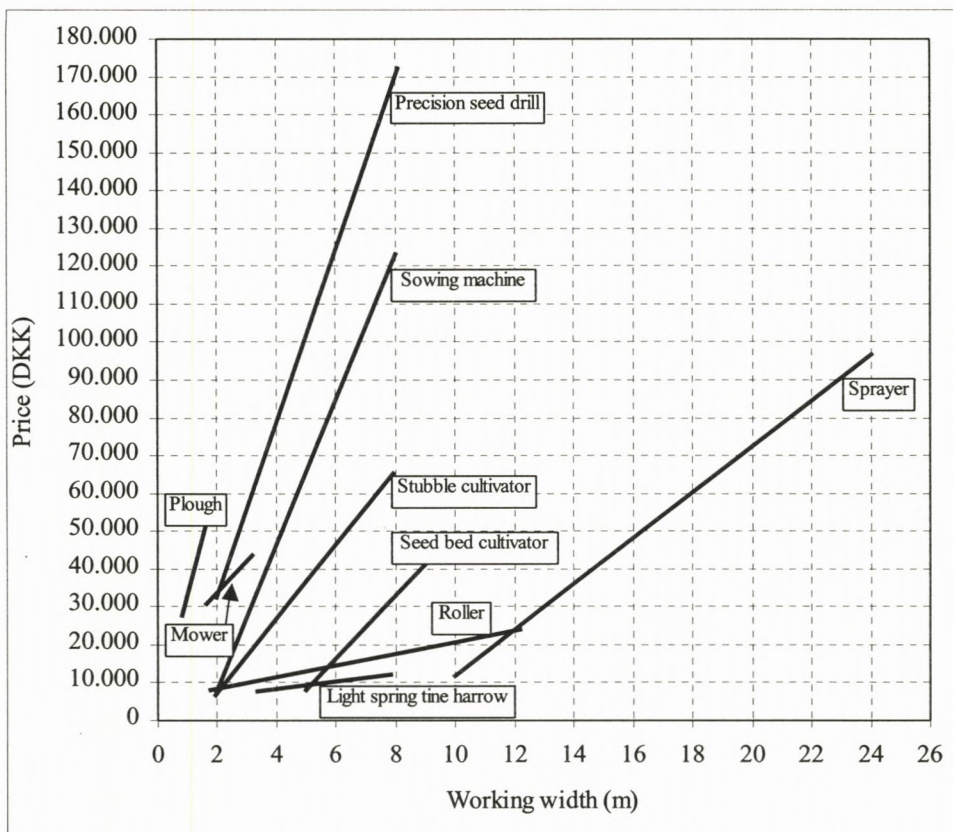


Figure 1. Prices of machinery based on working width (based on 1995 prices).

Table 2. Economical background data for the case study (based on 1995 conditions)

Quantity	Value
Interest rate	9%
Depreciation rate	12%
Labour cost	100 DKK/h
Purchase price of a tractor per unit of power	5.240 DKK/kW
Price of peas	1080 DKK/t
Price of peas as wholecrop for silage	378 DKK/t
Price of barley	870 DKK/t
Price of grass	286 DKK/t
Price of fodder beets	169 DKK/t
Price of rye	870 DKK/t
Price of wheat	890 DKK/t

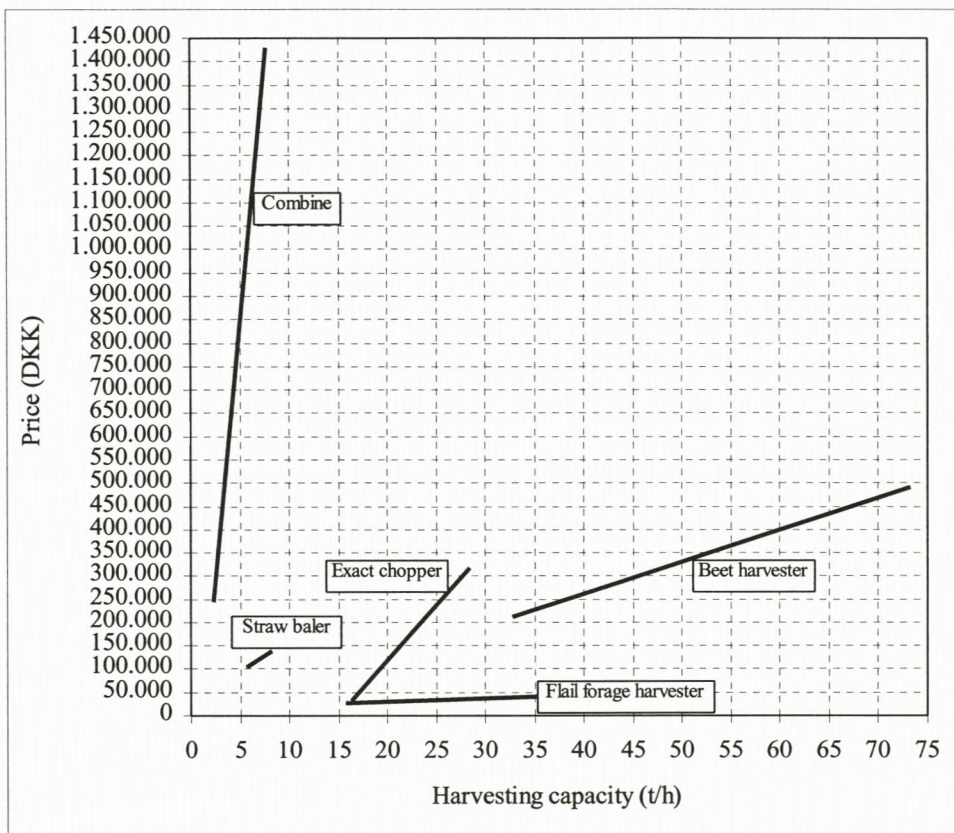


Figure 2. Prices of machinery based on harvesting capacity (based on 1995 prices).

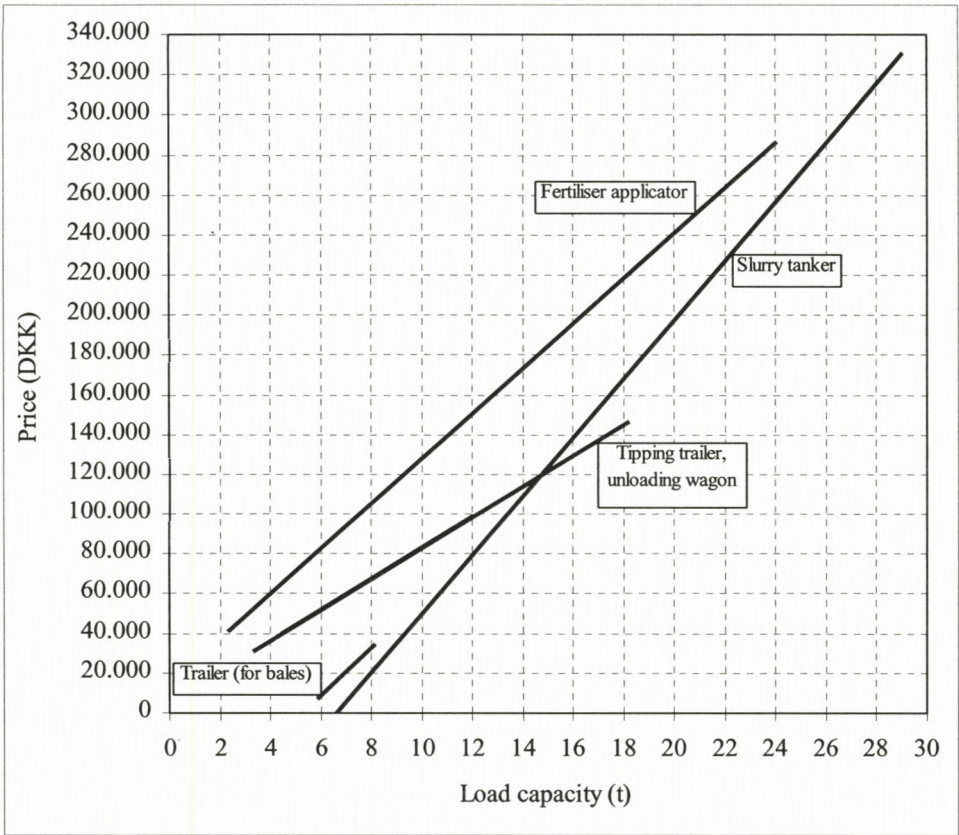


Figure 3. Prices of machinery based on load capacity (based on 1995 prices).

Table 3 shows some results from an optimisation of the machinery at the case farm. The table includes data from the actual machinery system present at the farm as well as data resulting from two optimisation runs. In the first optimisation run only the machinery types actually present at the case farm have been included. In the second optimisation run a combine and an exact chopper have been added to the machinery system.

For the sake of simplicity, the optimisation model operates with only one tractor size. In practice, only the largest tractor should have the size found by the optimisation model, while the remaining tractors may be less depending on the machinery at the farm. It appears from Table 3 that the optimal tractor size found in optimisation run 1 is almost the same as the actual size of the largest tractor on the case farm. In optimisation run 2 a larger tractor is optimal, due to the introduction of an exact chopper which requires more tractor power. Furthermore an increase of the tractor size will in general give rise to larger machinery which can utilise the increased tractor power. This, for instance, applies to machinery used in connection with seed bed preparation and sowing of grain in the spring (plough, light spring tine harrow, seed bed

cultivator, sowing machine and roller). Also notice that the optimal sizes of machinery for these operations have a tendency to be larger than the actual machines on the case farm. This fact underlines that seed bed preparation in the spring is critical and requires high machinery capacity.

In the case considered, the availability of labour has proven to be critical. During the early spring (weeks 12-14), the period of harvesting winter barley and wholecrop (weeks 28-31) and in late autumn (weeks 42-48) the upper limit of available man-hours with workable conditions has been reached (see Figure 4). Other things being equal, this circumstance will have an increasing effect on the optimal machinery sizes. The reason is that during periods where multiple operations “compete for” the same number of man-hours, the timeliness costs can only be limited by increasing the machinery sizes, as this will reduce the duration of the operations. To verify this hypothesis, an optimisation run with an unlimited number of man-hours has been performed. As expected, this run has led to reduced sizes of the machinery.

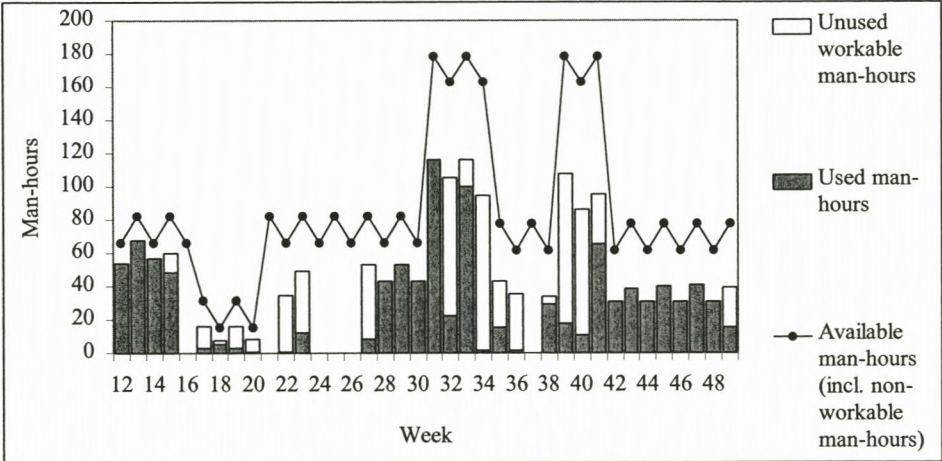


Figure 4. Used and unused man-hours when workability of crop and/or soil allows performance of field operations. The variation in available man-hours is due to holidays, the willingness to work overtime in peak load periods and the fact that one of the farm workers has every second weekend off.

As it appears from Table 3, the total annual costs associated with the optimal solution are rather high. In optimisation run 2, in which it has been assumed that the farm owns machinery for all operations, the total annual costs excluding timeliness costs amount to 491,897 DKK⁶. With an acreage of 81 ha this means that the total annual costs per hectare amount to 6,073 DKK, which is higher than average at Danish farms (1,500-6,000 DKK/ha/year). One of the

⁶ Monetary values given in Table 3 and mentioned in the text are based on 1995 prices.

reasons is that in this optimisation example it was assumed that all machines are new. Another reason is that all the tractors are assumed to be of the same size equal to the largest one. Both of these circumstances lead to some overestimation of the fixed costs. The assumption that the machinery system includes a combine and an exact chopper, although the farm is probably too small for that, also gives rise to increasing costs.

To compare the total annual costs in optimisation runs 1 and 2, the costs involved in run 1 should be increased by the expenses for the contractor work which can be estimated to about 80,000 to 90,000 DKK. By adding these expenses, the costs in optimisation run 1 rise to about 470,000-480,000 DKK which is still less than the costs in optimisation run 1. This means that it will be more profitable to the case farm to hire a contractor than to have own combine and exact chopper.

In general it can be concluded that the optimisation model gives sensible results, and the differences between the optimal machinery sizes and the actual ones are explicable on the basis of the conditions and data underlying the optimisation runs.

Table 3. Results from optimisation of the case farm machinery (based on 1995 prices)

Machinery/costs	Size of machinery			Unit
	Actual	Optimisation runs ^(*)		
		1	2	
Tractor 1	34	60.0	63.6	kW
Tractor 2	54	60.0	63.6	kW
Tractor 3	59	60.0	63.6	kW
Conventional plough	1.05	1.4	1.5	m
Light spring tine harrow	5.0	6.7	7.1	m
Seed bed cultivator	5.0	5.6	6.2	m
Stubble cultivator	4.8	2.1	2.1	m
Sowing machine	4.0	3.4	3.8	m
Roller	5.0	6.7	7.1	m
Sprayer	12.0	10.0	10.0	m
Fertiliser applicator	4.5	2.4	2.4	t
Mower	1.65	3.2	3.2	m
Exact chopper	N/A ^(**)	N/A ^(**)	21.1	t/h
Tipping trailer	4.5	3.4	3.9	t
Flail forage harvester	20	16.0	16.0	t/h
Beet harvester	26	33.0	33.0	t/h
Unloading wagon 1	6.5	5.9	5.9	t
Unloading wagon 2	3.5	5.9	5.9	t
Slurry tanker	6.0	6.0	6.4	t
Straw baler	7	5.9	5.9	t/h
Trailer (for bales)	?	2.5	2.5	t
Combine	N/A ^(**)	N/A ^(**)	2.3	t/h
Precision seed drill	2.5	2.0	2.0	m
Fixed costs		241,263	308,293	DKK
Operating costs, excl. labour costs		47,517	79,744	DKK
Labour costs		75,800	103,860	DKK
Timeliness costs		<u>24,049</u>	<u>24,748</u>	DKK
Total annual costs		388,629	516,645	DKK

(*) Optimisation run 1: only machinery actually present at the case farm is included.

Optimisation run 2: machinery covering all field operations at the case farm is included.

(**) N/A = Not available. Field operations performed by contractor.

4 Discussion and conclusion

When a mathematical model for optimisation of the size of farm machinery is formulated, several aspects should be considered. Timeliness and workability associated with the operations to be performed are important factors. Furthermore, it should be taken into consideration that the optimal machinery sizes are interdependent, since the individual machines make use of common resources, such as time, labour and tractors. Another important aspect is that the optimal farm machinery system is strongly connected with the cropping plan. If the plan changes significantly, then the optimisation procedure has to be repeated under the new conditions.

To formulate a model which can be implemented, it will be necessary to pay attention to the availability of data. On the other hand, the existing amount of data should not be considered too important when formulating the model. In several cases, the replacement of missing data by a good estimate is better than trying to reformulate the model so that the data will not be needed at all. However, the model study has shown that it would be desirable to get more thoroughly researched estimates of some of the data needed in the optimisation model, e.g. the timeliness and workability factors.

The GAMS model presented in this report has not been prepared with the user interface in mind. Consequently, it is rather time-consuming to run the model for a new farm, as the preparation of the input data files for the model is somewhat laborious. To make the model applicable to a wider category of users, it is therefore necessary to add an input generating module with a self-explanatory and interactive user interface. In this way, the effort required by the user can be minimised. Also, the output produced by the GAMS model might be put into a more user-friendly shell.

In its present form the GAMS model will find the optimal sizes of the machines listed by the user in a file for this purpose. However, the model is not able to make an optimal choice between different alternative types of machines. To implement this feature it would be necessary to introduce integer variables in the model, e.g. binary variables, which will indicate whether or not the alternative machines are included in the optimal machinery system for the farm in question. The introduction of integer variables would involve a transformation of the model from a non-linear programming model to a mixed integer non-linear programming model, which is far more difficult to solve (actually the solver used together with GAMS in the present study should be replaced to accomplish this task). However, an extension of the model with machine selection capabilities would make it possible to

- choose optimally between machinery alternatives for the same operation (e.g. conventional plough versus reversible plough)

- optimise the use of contractor work in order to avoid expensive machinery at the farm
- choose optimally between new and used machinery.

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Appendix A

Calculation of α , β , γ , δ and s for some machinery types and operations

The definition of the parameters α , β , γ , δ and s , which are introduced through the optimisation model described in Section 2.3, depend on the machinery considered⁷. In the following sections it will be described how the parameters are calculated for individual machines and whole machinery sets (operations).

The meaning of the parameters can be described briefly as follows:

- α is a parameter related to the fuel, repair and maintenance costs of the machinery,
- β is a parameter related to labour costs and repair/maintenance costs of the machinery,
- γ is a parameter related to repair and maintenance costs of the tractor(s),
- δ is a timeliness factor related to the operation,
- s is a coefficient which is used for conversion of the size of a machine to its effective capacity in a given operation.

A.1 Calculation of α , β , γ and s for individual machines

Table 4 shows how to calculate α , β , γ and s for some machinery types. The meaning of various quantities introduced in the table appears from Table 5. As it can be seen from Table 4, the calculation formulas for α , β , γ and s depend on the quantity used for characterisation of the size of the machinery type (theoretical working width, theoretical harvesting capacity or load capacity). This means that calculation formulas for machinery types not mentioned in the table can easily be added, once it has been determined which of these quantities characterise its size most suitably.

⁷ Strictly speaking, α , β , γ , δ and s should be accompanied with indices in the following way: α_j , β_j , γ_j , δ_j and $s_{j,i}$ ($i = 1, \dots, N^M$; $j = 1, \dots, N^O$). However, for the sake of simplicity, the indices have been left out in this appendix.

Table 4. Formulas for calculation of α , β , γ and s for some machinery types

Machinery type	Formulas	Units	Eff. capacity	Size
Plough, harrow, roller, sowing machine, sprayer, mower, precision seed drill	$\alpha = A(rp_{b1} + B_b)/(ve)$ $\beta = A(L + rp_{b0})$ $\gamma = Ad^{(*)}$ $s = 1/(ve)$	DKK (DKK m ²)/h (DKK m ²)/(h W) h/m	vbe [m ² /h]	Theoretical working width, b [m]
Combine, flail forage harvester, exact chopper, beet harvester, straw baler	$\alpha = AU(rp_{k1} + B_k)/e$ $\beta = AU(L + rp_{k0})$ $\gamma = AUd^{(*)}$ $s = 1/e$	DKK (DKK t)/h (DKK t)/(h W) Dimensionless	Ke [t/h]	Theoretical harvesting capacity, K [t/h]
Trailer, fertiliser spreader ^(**)	$\alpha = M\tau(rp_{m1} + B_m)$ $\beta = M(L + rp_{m0})$ $\gamma = Md^{(*)}$ $s = \tau$	DKK (DKK t)/h (DKK t)/(h W) h	m/τ [t/h]	Load capacity, m [t]

(*) If the machinery is self-propelled, then $\gamma = 0$.

(**) Fertiliser spreaders are placed in the same category as trailers, as the available data concerning fertiliser spreaders give the purchase price as a function of the hopper size (load capacity).

Table 5. Nomenclature

Symbol	[unit]	Description	m/o ^(*)
τ	[h]	Time used for one transportation cycle (from starting one loading until the next one is started).	o
A	[m ²]	Area to be "treated".	o
b	[m]	Theoretical working width.	m
B_b	[DKK/(m h)]	Fuel and oil expenses per operating hour and per unit of theoretical working width.	m, o
B_k	[DKK/t]	Fuel and oil expenses per operating hour and per unit of theoretical harvesting capacity.	m, o

Symbol	[unit]	Description	m/o ^(*)
B_m	[DKK/(t h)]	Fuel and oil expenses per operating hour and per unit of load capacity.	m, o
d	[DKK/(W h)]	Repair and maintenance costs of one tractor per unit of tractor power and per operating hour ($d = 0$ for self-propelled machines).	
e	Dimensionless	Field efficiency ($0 < e \leq 1$).	m, o
K	[t/h]	Theoretical harvesting capacity.	m
L	[DKK/h]	Labour costs.	o
m	[t]	Load capacity.	m
M	[t]	Total amount (mass) to be transported. If a trailer is used for transportation of a harvested crop, then $M = AU$.	o
p_{b0} p_{b1}	[DKK] [DKK/m]	Y intercept and slope of the linear equation which expresses the purchase price as a function of the theoretical working width.	m
p_{k0} p_{k1}	[DKK] [DKK/(t/h)]	Y intercept and slope of the linear equation which expresses the purchase price as a function of the theoretical harvesting capacity.	m
p_{m0} p_{m1}	[DKK] [DKK/t]	Y intercept and slope of the linear equation which expresses the purchase price as a function of load capacity.	m
r	[h ⁻¹]	Repair and maintenance costs per operating hour, expressed as a fraction of the purchase price.	m
R	[uge ⁻¹]	Timeliness costs per week if the operation is performed before or after the time of optimum crop return (quantity and quality).	o
U	[t/m ²]	Crop yield. In cereal crops U is the straw yield (or in general, the yield fraction that occupies processing capacity of the harvesting machine).	o

Symbol	[unit]	Description	m/o ^(*)
U_v	[t/m ²]	Crop yield if all operations related to the crop have been performed at such a time that crop return is optimised considering quality and quantity of product. In cereal crops U_v is the grain yield.	o
v	[m/h]	Field speed.	m, o
V	[DKK/t]	Expected value of the crop at harvest time.	o

^(*) Symbols used in the column "m/o":

"m" – The quantity in the first column applies to a machine.

"o" – The quantity in the first column applies to an operation.

"m, o" – The quantity in the first column applies to the combination of a machine and an operation.

A.2 Calculation of α , β and γ for an operation

Table 6 shows how to calculate α , β and γ for a given operation based on α , β and γ for the individual machines in the machinery set which is used for the performance of this particular operation (see Section A.1).

Table 6. Methods for calculation of α , β and γ for machinery sets

Parameter	Parallel operation ^(*)	Serial operation ^(**)
α	Sum of α 's for individual machines	Sum of α 's for individual machines
β	Sum of β 's for individual machines	Average of β 's for individual machines
γ	Sum of γ 's for individual machines	Average of γ 's for individual machines

^(*) The machines in the machinery set are operating simultaneously.

^(**) The machines in the machinery set are operating by turns.

Ideally, the validity of the calculation methods shown for β and γ in the table implies that the individual machines in the machinery set are harmonized with respect to capacity. Since this condition will not be fulfilled in the general case, the calculation methods can be regarded as approximations. The idea of assuming harmonized machines when calculating machinery costs for a set of machines is described by Have (1991). Notice that the calculation of the sums and averages for β and γ presupposes that the effective capacities of individual machines in the machinery set are of the same unit.

A.3 Calculation of δ for an operation

The timeliness factor δ [DKK/week] associated with an operation should be calculated as follows:

$$\delta = RAU_p V$$

The quantities used on the right hand side of this equation is explained in Table 5. In case an operation does not involve a direct timeliness effect, then $\delta = 0$ should be applied, since $R = 0$. Consider for instance the operations connected with a cereal crop which is sown in spring. Among other things, the operations include harrowing, sowing and harvesting. Among those operations, only sowing and harvesting involve a direct timeliness effect: sowing, because delayed sowing will lead to a reduced growing season and thus a reduction of crop return, and harvesting, because harvesting before or after the optimum time as to maturity will lead to reduced crop return. On the other hand, delayed harrowing does not involve a direct timeliness effect, but only an indirect timeliness effect through delayed sowing.

A.4 Calculation of s

In most cases the s -values can be calculated as shown in Table 4. However, if two or more machines are taking part in accomplishing the same subtask within an operation, s should be calculated in a different way. If, for instance, m machines, e.g. combines, are operating simultaneously on the same field, the following calculation method should be used:

$$s_{\text{Machine } i} = \frac{1}{m} s_{\text{Table 4}}, \quad i = 1, \dots, m$$

This formula is an approximation, since it implies that the work is shared equally among the m machines.

Appendix B

Limitations on man-hours, machine-hours and tractor-hours

The number of operations performable during a given week is, among other things, limited by the workability of soil and/or crop and the availability of man-hours, machinery-hours and tractor-hours. This appendix describes how these limiting factors have been taken into account in the optimisation model (see (2), (3) and (4) on page 13).

B.1 Modelling limitations on man-hours

In the following paragraphs the limitations on man-hours in a given week, k ($k = 1, \dots, 52$), are considered. The following quantities are introduced:

- T'_k is the working hours which can potentially be used for field work during week k ,
- T_k is the man-hours available for field work within the potential field working hours (T'_k) in week k ,
- n is the number of operations which should be performed⁸ during week k ,
- j is used for numbering the operations ($j = 1, \dots, n$),
- w_j is the workability factor which is defined as the fraction of the potential field working hours which is left for the performance of the operation when the expected hours with unfavourable climate, soil or crop conditions have been left out ($0 \leq w_j \leq 1$),
- $t_{j,k}$ is the effective duration of operation j (corresponds to the expression in (19), page 18),
- r_j is the number of workers involved full-time in the performance of operation j .

From the above definitions it can be seen that $r_j t_{j,k}$ expresses the number of man-hours which is necessary to complete operation j .

Concerning T'_k it should be noticed that this quantity is the potential number of field working hours before deduction of hours where soil and/or crop is not workable. Therefore, T'_k is normally greater than the actual number of hours which can be used for field work during week k . The same circumstance applies to T_k .

To simplify the model formulation, the following three assumptions have been made:

⁸ In fact, only a certain part (0-100%) of each operation should be performed during week k . However, for simplification of the description in this appendix, the term "operation" is used where the meaning is "the part of operation performed during week k ".

1. *Each operation is performed with uniform intensity (measured as man-hours used per real hour) over the period when workability makes the operation possible. Thus, it is assumed that when a certain percentage of the “workable time” associated with a given operation has passed, then the same percentage of the operation will have been completed. This is, of course, not a fully realistic assumption, as the consequence would be that two or more operations might be going on at the same time (parallel performance) in situations where it would be more natural to finish one operation before the next one is started (sequential performance). However, if the w_j -values are equal across operations, the model formulation would end up the same, whether the operations are assumed to be performed in parallel or in sequence. On the other hand, if the w_j -values are very different, the assumption of uniform performance intensity will lead to non-optimal utilisation of the available number of man-hours, thus having a certain tendency to make the model overestimate the optimal machine sizes.*
2. *If a given operation, j_A , makes larger demands on workability than another operation, j_B , i.e. $w_{j_A} < w_{j_B}$, then the hours when operation j_A is possible will be a subset of the hours when operation j_B is possible.*
3. *The man-hours available for field work in week k are uniformly distributed over the potential field working hours in week k .*

Figure 5 illustrates the situation when the available man-hours should be shared among multiple operations.

Operation j	Hours in week k when workability makes performance of operation j possible		Man-hours used for the performance of operation j in week k
1		$w_1 T'_k$	$r_1 t_{1,k}$
2		$w_2 T'_k$	$r_2 t_{2,k}$
\vdots		\vdots	\vdots
j^*		$w_{j^*} T'_k$	$r_{j^*} t_{j^*,k}$
\vdots		\vdots	\vdots
n		$w_n T'_k$	$r_n t_{n,k}$
	Critical hours		
	T'_k (= potential field working hours in week k)		

Figure 5. Sharing of man-hours among multiple operations.

The most critical hours during the week are when the workability makes all n operations possible. This is because all operations are assumed to be going on parallel with each other during these hours (due to assumption 1). In consequence of assumption 2, the critical hours will coincide with the hours when the operation with largest demands on workability is possible. Assume that this operation is number j^* (see Figure 5). The number of critical field working hours is $w_{j^*} T'_k$, while the number of field working hours over which operation j is performed (with uniform intensity) is $w_j T'_k$. This means that the fraction of operation j which is completed within the critical hours is

$$\frac{w_{j^*} T'_k}{w_j T'_k} = \frac{w_{j^*}}{w_j}$$

By multiplying this fraction by the total number of man-hours used for operation j which is $r_j t_{j,k}$, the number of man-hours used for the operation within the critical hours can be achieved as follows:

$$\frac{w_{j^*}}{w_j} r_j t_{j,k}$$

The total number of man-hours used by all the operations within the critical hours is found by summing up this expression over all operations:

$$\sum_{j=1}^n \frac{w_{j^*}}{w_j} r_j t_{j,k} = w_{j^*} \sum_{j=1}^n \frac{r_j t_{j,k}}{w_j}$$

Because of assumption 3 the maximum number of man-hours available during the critical hours is calculated as $w_{j^*} T_k$. Consequently, the following inequality must be satisfied:

$$w_{j^*} \sum_{j=1}^n \frac{r_j t_{j,k}}{w_j} \leq w_{j^*} T_k$$

or after elimination of w_{j^*} :

$$\sum_{j=1}^n \frac{r_j t_{j,k}}{w_j} \leq T_k \quad (20)$$

Since the quantity $t_{j,k}$ corresponds to the expression in (19) on page 18, it will be seen that the inequality in (20) corresponds to the constraint given in (2) on page 13.

B.2 Modelling limitations on machine-hours

The limitations on machine-hours is basically handled in the same way as limitations on man-hours. Therefore, the assumptions put forward in Section B.1 have also been adopted in this section. Below, the limitations on machine-hours for a given machine in a given week, k ($k = 1, \dots, 52$), are considered. The following quantities are introduced:

- T^w is the total number of working hours available during one week,
- n is the number of operations in which the machine is used in week k ,
- j is used for numbering the operations in week k for which the machine is used ($j = 1, \dots, n$),

w_j is the workability factor ($0 \leq w_j \leq 1$),
 $t_{j,k}$ is the effective duration of operation j .

The fact that the number of used machine-hours must not exceed the maximum number of working hours (T^w) leads to the following restriction (using arguments which is similar to those put forward in Section B.1):

$$\sum_{j=1}^n \frac{t_{j,k}}{w_j} \leq T^w \quad (21)$$

Since the quantity $t_{j,k}$ corresponds to the expression in (19) on page 18, it can be seen that the inequality in (21) corresponds to the constraint given in (3) on page 12.

B.3 Modelling limitations on tractor-hours

The assumptions and the approach described in Section B.1 are also employed when considering limitations on tractor-hours. Below, the limitations on tractor-hours in a given week, k ($k = 1, \dots, 52$), are considered. The following quantities are introduced:

T^w is the total working hours available during one week,
 n is the number of operations which should be performed during week k ,
 N is the number of tractors on the farm,
 j is used for numbering of the operations ($j = 1, \dots, n$),
 w_j is the workability factor ($0 \leq w_j \leq 1$),
 $t_{j,k}$ is the effective duration of operation j ,
 q_j is the number of tractors involved full-time in the performance of operation j .

The fact that the number of used tractor-hours must not exceed the maximum number of tractor-hours (NT^w) leads to the following restriction (using arguments which are similar to those put forward in Section B.1):

$$\sum_{j=1}^n \frac{q_j t_{j,k}}{w_j} \leq NT^w \quad (22)$$

Because the quantity $t_{j,k}$ corresponds to the expression in (19) on page 18, it can be seen that the inequality in (22) corresponds to the constraint given in (4) on page 13.

Appendix C

Listing of the GAMS model code

This appendix shows a listing of the file containing the main GAMS model code which implements the non-linear optimisation model described in Chapter 2. The data, which are also a part of the model, do not appear from the program listing. The reason is that the data have been isolated in separate data files, which are included in the GAMS model code using GAMS INCLUDE statements. The structure of the data files are described in Appendix D.

In the GAMS model code it has been assumed that a so-called CONOPT options file is present in the same directory as the GAMS file itself. The CONOPT options file is a plain ASCII file named CONOPT . OPT. The file should only contain a single line with the following text:

```
SET RTMAXJ 1.00E+15
```

This line ensures that Jacobian elements not exceeding 10^{15} are allowed in the non-linear programming algorithm CONOPT which is used for solution of the optimisation model.

The following pages show the program listing.


```

$TITLE Model for Optimisation of the Size of Farm Machinery
$EOLCOM !
* =====
*
* This GAMS program source file implements a non-linear model for
* optimisation of machinery sizes in a farm machinery system.
*
* Programmer: Henning T. Soegaard
*
* Date: 14 May 1998
*
* =====

* =====
* Declaration and definition of SETS
* =====

* ----- Declaration and static definition of SETS -----

SETS
    I      "Machines in the farm machinery system"
        /
$        INCLUDE "machines.inc"
        /

    J      "Operations to be performed during the year"
        /
$        INCLUDE "operatio.inc"
        /

    K      "Week numbers"
        /W01*W52/

    JXI(J,I) "Machines (I) used for the performance of operation (J)"
        /
$        INCLUDE "permach.inc"
        /

    JXK(J,K) "Week numbers (K) in which operation (J) is performable"
        /
$        INCLUDE "operweek.inc"
        /

    KXI(K,I) "Machines (I) which may be used in varies week (K)"

    ODATNAME "Parameter names of data required for each operation"
        /
        ALPHA, BETA, GAMMA, DELTA "Param. associated with operating costs"
        TOPT      "Week number with min. timeliness costs"
        A          "Field area (m2)"
        U          "Crop yield (ton/m2 or dimensionless)"
        R, Q       "Number of workers and tractors"
        W          "Workability factor"

```

```

/

MDATNAME "Parameter names of data required for each machine"
/
FI0  "Y-intercept for calc. of fixed costs from machine size"
FI1  "Slope for calc. of fixed costs from machine size"
THETA "Prop. factor for calc. of min. tractor power from mach. size"
XMMIN "Minimum available machine size"
XMMAX "Maximum available machine size"
/

OTYPE "Names of types of operations"
/
PARALLEL "The machines are operating simultaneously"
SERIAL   "The machines are operating by turns"
/

OPERTYPE(J,OTYPE) "Type of each operation"
/
$ INCLUDE "opertype.inc"
/ ;

ALIAS (K, KK) ; ! "KK is an auxiliary index used in EQUATIONS definitions"

ALIAS (J, JJ) ; ! "JJ is an auxiliary index used in EQUATIONS definitions"

ALIAS (J, JF) ; ! "JF are operations to be performed before operation J"

SET
  JXJF(J,JF) "Operations (JF) to be performed before operation J"
/
$ INCLUDE "operseq.inc"
/ ;

* ----- Dynamic definition of SETS -----

XXI(K,I) = YES $ SUM(J, JXI(J,I)*JXK(J,K)) ;

* =====
* Declaration and definition of SCALARS, PARAMETERS and TABLES
* =====

* ----- Declaration of SCALARS -----

SCALARS
  CT  "Fixed annual costs for tractor expr. as fraction of purchase price"

  PT  "Price per power unit at purchase of own tractor (DKK/W)"

  PSI "CT*PT (DKK/W)"

  TW  "Total working hours during one week" ;

```

```

* ----- Dynamic definition of SCALARS -----

$INCLUDE "miscdata.inc"
PSI = CT*PT ;

* ----- Declaration and static definition of PARAMETERS -----

PARAMETERS
    T(K) "Man-hours available for field work in various weeks"
    /
$    INCLUDE "manhour.inc"
    /

    ALPHA(J) "Parameter associated with operating costs of operation J"

    BETA(J) "Parameter associated with operating costs of operation J"

    GAMMA(J) "Parameter associated with operating costs of operation J"

    DELTA(J) "Parameter associated with timeliness costs of operation J"

    TOPT(J) "Week number when timeliness costs of operation J is minimum"

    A(J) "Field area (m2) to be treated in operation J"

    U(J) "Expected crop yield (ton/m2 or dimensionless) in operation J"

    R(J) "Number of workers engaged in operation J"

    Q(J) "Number of tractors used in operation J"

    W(J) "Timeliness factor for operation J"

    FI0(I) "Y-intercept for calc. of fixed costs from size of machine I"

    FI1(I) "Slope for calc. of fixed costs from size of machine I"

    THETA(I) "Prop. fact. for calc. of min. tract. pwr. from size of mach. I"

    XMMIN(I) "Minimum available size of machine type I"

    XMMAX(I) "Maximum available size of machine type I"

    AU_W(J) "PARAMETER for simplification of EQUATION-defs. (ton or m2)";

* ----- Declaration and definition of TABLES -----

TABLE
    ODATA(J,ODATNAME) "Data per operation"
$    INCLUDE "operdata.inc"
    ;

```


TABLE

```

      MDATA(I,MDATNAME) "Data per machine"
$      INCLUDE "machdata.inc"
;

```

* Note: Tables ODATA and MDATA are exclusively introduced to enable
 * reading of operation and machinery specific data as tables from
 * INCLUDE files. For the sake of readability of the EQUATION
 * definitions the data from ODATA and MDATA is transferred to
 * individual parameters (see below) which is used in the EQUATION
 * definitions.

PARAMETER

```

      S(J,I) "Factor for conv. of size of mach. I to eff. capacity of oper. J"
      /
$      INCLUDE "capfac.inc"
      /
;

```

* ----- Dynamic definition of PARAMETERS -----

```

ALPHA(J) = ODATA(J,"ALPHA") ;
BETA(J)  = ODATA(J,"BETA") ;
GAMMA(J) = ODATA(J,"GAMMA") ;
DELTA(J) = ODATA(J,"DELTA") ;
TOPT(J)  = ODATA(J,"TOPT") ;
A(J)     = ODATA(J,"A") ;
U(J)     = ODATA(J,"U") ;
R(J)     = ODATA(J,"R") ;
Q(J)     = ODATA(J,"Q") ;
W(J)     = ODATA(J,"W") ;
FI0(I)   = MDATA(I,"FI0") ;
FI1(I)   = MDATA(I,"FI1") ;
THETA(I) = MDATA(I,"THETA") ;
XMMIN(I) = MDATA(I,"XMMIN") ;
XMMAX(I) = MDATA(I,"XMMAX") ;
AU_W(J)  = A(J)*U(J)/W(J) ;

```

* =====
 * Declaration of VARIABLES and assignment of their .L-, .LO- and .UP-values
 * =====

* ----- Declaration of VARIABLES -----

FREE VARIABLE

```

F      "Total annual costs (DKK)"

```

POSITIVE VARIABLES

```

XO(J)  "Eff. cap. of machinery set used for oper. J (ton/h or m2/h)"
X(J,K) "Fraction of operation J performed in week K"
XM(I)  "Size of machine I (ton/h, m or ton)"

```

```

      XT      "Tractor power (W)"
      ZT      "Number of tractors" ;

* Note: Ideally ZT should have been declared as an INTEGER VARIABLE but as a
*       MINLP SOLVER is not available, ZT is declared as a non-negative real
*       variable to allow use of a NLP SOLVER.

* ----- Assignment of .L-, .LO- and .UP-values to VARIABLES -----

* -- XM.LO and XM.UP is takes naturally the values XMIN and XMAX,
* -- respectively, while XMMAX is assigned to XM.L to try ensuring that the
* -- initial solution is feasible (see remarks on assignment of value to
* -- XO.L below).
XM.LO(I) = XMMIN(I) ;
XM.UP(I) = XMMAX(I) ;
XM.L(I)  = XMMAX(I) ;

* -- A small number (1.0E-8, not zero) is assigned to XO.LO to avoid division
* -- by zero in the EQUATIONS where division by XO takes place. The value
* -- assigned to XO.L is the largest possible which ensures that the
* -- constraints CAPRELATI1 and CAPRELATI2 are satisfied (see below). As XM.L
* -- takes the value XMMAX, XO.L and XM.L will take the largest possible
* -- values which ensure that CAPRELATI1 and CAPRELATI2 is satisfied. Since
* -- division by XO takes place in the EQUATIONS MANHOURS, MACHHOURS og
* -- TRACTHOURS these EQUATIONS will most likely be satisfied.
XO.LO(J) = 1.0E-8 ;
XO.L(J) $ OPERTYPE(J,"PARALLEL") = SMIN(I $ (S(J,I) NE 0), XM.L(I)/S(J,I)) ;
XO.L(J) $ (NOT OPERTYPE(J,"PARALLEL")) = 1/SUM(I, S(J,I)/XM.L(I)) ;

* -- Since X is declared as a POSITIVE VARIABLE and the X values must add to
* -- 1 for each operation (see the EQUATION FINISHOPER) the constraint
* -- 0 <= X.L <= 1 will automatically be satisfied. Therefore it is not
* -- necessary to assign values to X.LO and X.UP.
* -- X.L(J,K) takes the value 1 if operation J if timeliness costs are
* -- minimum in week K and 0 otherwise.
X.L(J,K) $ ((ORD(K) EQ TOPT(J)) $ JXK(J,K)) = 1.0 ;

* -- The value assigned to XT.L is the maximum tractor power required by any
* -- machine given the values assigned to XM.L above. This is done to ensure
* -- that the EQUATION POWERCAP (see below) is satisfied initially.
XT.L      = SMAX(I, THETA(I)*XM.L(I)) ;

* -- The number of tractors must be greater than or equal to the number
* -- used in the operation requiring most tractors to operate simultaneously.
* -- This is why ZT.L takes the maximum value of Q(J) over J. The value
* -- assigned to ZT.L is either the value of ZT.LO or the least value of ZT.L
* -- which meets the requirement in the EQUATION TRACTHOURS in all weeks. The
* -- maximum of these two values are assigned to ZT.L.
ZT.LO     = SMAX(J, Q(J)) ;
ZT.L      = MAX( SMAX(K, SUM(J, Q(J)*X.L(J,K)*AU_W(J)/XO.L(J)))/TW, ZT.LO) ;

* =====
* Declaration and definition of EQUATIONS
* =====

```

* ----- Declaration of EQUATIONS -----

EQUATIONS

OMKOST "Define the objective function (total annual costs)"
 MANHOURS(K) "Available man-hours in week K is limited"
 MACHHOURS(K,I) "Available hours for machine I in week K is limited"
 TRACTHOURS(K) "Available tractor-hours in week K is limited"
 FINISHOPER(J) "Operations must be completed 100%"
 OPERSEQUEN(J,JF,K) "Operations must be performed in proper sequence"
 CAPRELATI1(J,I) "Relation between eff. cap. and mach. size, parallel"
 CAPRELATI2(J) "Relation between eff. cap. and mach. size, serial"
 POWERCAP(I) "Relation between mach. size and needed tractor power";

* ----- Definition of EQUATIONS -----

OMKOST ..

F =E= SUM(I, FII(I)*XM(I) + FIO(I)) + PSI*ZT*XT
 + SUM((K,J) \$ JXK(J,K),
 X(J,K)*(ALPHA(J) + (BETA(J) + GAMMA(J)*XT)/XO(J)
 + DELTA(J)*ABS(ORD(K) - TOPT(J)))) ;

MANHOURS(K) \$ SUM(J, JXK(J,K)) ..

SUM(J \$ JXK(J,K), R(J)*X(J,K)*AU_W(J)/XO(J)) =L= T(K) ;

MACHHOURS(K,I) \$ KXI(K,I) ..

SUM(J \$ (JXI(J,I)*JXK(J,K)), X(J,K)*AU_W(J)/XO(J)) =L= TW ;

TRACTHOURS(K) \$ SUM(J, JXK(J,K)) ..

SUM(J \$ JXK(J,K), Q(J)*X(J,K)*AU_W(J)/XO(J)) =L= ZT*TW ;

FINISHOPER(J) ..

SUM(K \$ JXK(J,K), X(J,K)) =E= 1 ;

OPERSEQUEN(J,JF,K) \$ (JXJF(J,JF)*JXK(J,K)*JXK(JF,K)) ..

SUM(KK \$ (ORD(KK) LE ORD(K)), X(JF,KK)\$JXK(JF,KK))
 =G=
 SUM(KK \$ (ORD(KK) LE ORD(K)), X(J,KK) \$JXK(J,KK)) ;

CAPRELATI1(J,I) \$ (JXI(J,I)*OPERTYPE(J,"PARALLEL")) ..

XO(J) =L= XM(I)/S(J,I) ;

CAPRELATI2(J) \$ (NOT OPERTYPE(J,"PARALLEL")) ..

XO(J) =L= 1/SUM(I \$ JXI(J,I), S(J,I)/XM(I)) ;

POWERCAP(I) ..

XT =G= THETA(I)*XM(I) ;

* =====
 * Declaration and definition of MODELs
 * =====

MODEL


```

MACHOPTIM
/OMKOST, MANHOURS, MACHHOURS, TRACTHOURS, FINISHOPER,
OPERSEQUEN, CAPRELATI1, CAPRELATI2, POWERCAP / ;

* =====
* SOLVE STATEMENTS
* =====

OPTION NLP = CONOPT ;

MACHOPTIM.OPTFILE = 1;
SOLVE MACHOPTIM USING NLP MINIMIZING F ;

* =====
* Post processing of the model solution and writing of results to disk file.
* =====

* ----- Declaration and static definition of SETS -----

SET
    SIZEUNIT "Units used for size of machines"
        /
        METRE
        TONSPRHOOR
        TONS
        /

    MACHUNIT(I,SIZEUNIT) "Unit used for size of machine I"
        /
$    INCLUDE "machunit.inc"
        / ;

* ----- Declaration of PARAMETERS -----

PARAMETERS
    FIXEDCOST          "Fixed annual costs"
    OPRCOST             "Annual operating costs"
    TIMELICOST          "Timeliness costs"
    WEEKNUMBER(K)       "Week number of week K"
    T_SUM               "Available man-hours during one year"
    T_PCT(K)            "Available man-hours in week K (percent)"
    WRKPROFILE(K)       "Use of man-hours in week K"
    WRKREST(K)          "Unused available man-hours in week K"
    WRKPRF_SUM          "Annual consumption of man-hours"
    WRKRST_SUM          "Unused available man-hours annually"
    WRKMARG(K)          "Marg. cost reduct. (DKK) by extra man-hours in week K"
    WRKPRF_PCT(K)       "Use of man-hours in week K (percent)"
    WRKRST_PCT(K)       "Unused available man-hours in week K (percent)"
    WRKWTUSED(k)        "Use available man-hours with workability (percent)"
    MACH_PCT(K,I)       "Percentage utilisation of machine I in week K"
    TRACTO_PCT(K)       "Percentage utilisation of tractors in week K"

```

```

TRACTUTIL(I)      "Utilisation (%) of tractor power for machine I"
OPERPLAN(J,K)     "Percentage of operation J performed in week K"
TRACTPOWER        "Tractor power (kW)"
HAPERHOUR(J)      "XO(J) converted to ha/hour where relevant" ;

* ----- Calculation of PARAMETER values -----

FIXEDCOST
  = SUM(I, FI1(I)*XM.L(I) + FI0(I)) + PSI*ZT.L*XT.L ;

OPRCOST
  = SUM((K,J) $ JXK(J,K),
        X.L(J,K)*(ALPHA(J) + (BETA(J) + GAMMA(J)*XT.L)/XO.L(J))) ;

TIMELICOST
  = SUM((K,J) $ JXK(J,K), X.L(J,K)*DELTA(J)*ABS(ORD(K) - TOPT(J))) ;

WEEKNUMBER(K) $ SUM(J, JXK(J,K))
  = ORD(K) ;

T_SUM
  = SUM(K $ SUM(J, JXK(J,K)), T(K)) ;

T_PCT(K)
  = 100 $ (T(K) NE 0) + NA $ (T(K) EQ 0) ;

WRKPROFILE(K) $ SUM(J, JXK(J,K))
  = SUM(J $ JXK(J,K), R(J)*X.L(J,K)*A(J)*U(J)/XO.L(J)) ;

WRKREST(K) $ SUM(J, JXK(J,K))
  = T(K) - WRKPROFILE(K) ;

WRKPRF_SUM
  = SUM(K, WRKPROFILE(K)) ;

WRKRST_SUM
  = SUM(K, WRKREST(K)) ;

WRKMARG(K) $ SUM(J, JXK(J,K))
  = MANHOURS.M(K) ;

WRKPRF_PCT(K) $ (T(K) NE 0)
  = WRKPROFILE(K)/T(K)*100 ;
WRKPRF_PCT(K) $ (T(K) EQ 0)
  = NA ;

WRKRST_PCT(K) $ (T(K) NE 0)
  = WRKREST(K)/T(K)*100 ;
WRKRST_PCT(K) $ (T(K) EQ 0)
  = NA ;

WRKWTUSED(K) $ (SUM(J, JXK(J,K))*(YES$(T(K) NE 0)))
  = MANHOURS.L(K)/T(K)*100 ;

```

```

WRKWITHUSED(K) $ (SUM(J, JXK(J,K))*(YES$(T(K) EQ 0)))
    = NA ;

MACH_PCT(K,I)
    = MACHHOURS.L(K,I)/TW*100 ;

TRACTO_PCT(K)
    = (1 + TRACTHOURS.L(K)/(ZT.L*TW))*100 ;

OPERPLAN(J,K)
    = 100*X.L(J,K) ;

TRACTPOWER
    = XT.L/1000 ;

HAPERHOUR(J) $ (XO.L(J) GE 1000)
    = XO.L(J)/10000 ;

* ----- Writing results to disk file -----

FILE RES /result.txt/ ;          ! "Establish reference to physical disk file"
PUT RES ;                        ! "Open file for write access"

PUT "|=====|" / ;
PUT "|    RESULTS FROM SOLVING NON-LINEAR MODEL FOR    |" / ;
PUT "|    OPTIMISATION OF THE SIZE OF FARM MACHINERY    |" / ;
PUT "|=====|" / ;

PUT / "Model status (see 'GAMS, A User's Guide'):" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 2) "Locally optimal" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 3) "Unbounded" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 4) "Infeasible" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 5) "Locally infeasible" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 6) "Intermediate infeasible" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 7) "Intermediate nonoptimal" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 12) "Error unknown" ;
PUT $ (MACHOPTIM.MODELSTAT EQ 13) "Error no solution" ;

PUT // "Solver status (see 'GAMS, A User's Guide'):" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 1) "Normal completion" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 2) "Iteration interrupt" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 3) "Resource interrupt" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 4) "Terminated by solver" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 5) "Evaluation error limit" ;
PUT $ (MACHOPTIM.SOLVESTAT EQ 6) "Unknown" ;
PUT $ (MACHOPTIM.SOLVESTAT GE 7) "Error ..." ;

PUT /// "VALUE OF OBJECTIVE FUNCTION" ;

PUT // "Total annual costs:    ", F.L:12:0, " DKK" ;
PUT /  "of this"
PUT /  "    fixed costs:      ", FIXEDCOST:12:0, " DKK" ;
PUT /  "    operating costs:  ", OPRCOST:12:0, " DKK" ;

```



```

PUT / "    timeliness costs: ", TIMELICOST:12:0, " DKK" ;

PUT /// "VALUES OF DECISION VARIABLES" ;

PUT // "Size of machines:" ;
PUT // "Machine          Size      Interval      Unit          Marg.price (DKK)" ;
LOOP (I,
  PUT / I.TL:12, XM.L(I):10:1, XM.LO(I):6:1, " - ", XM.UP(I):5:1 ;
  PUT $ MACHINUNIT(I,"METRE")      " metre " ;
  PUT $ MACHINUNIT(I,"TONSPRHOURL") " ton/hour" ;
  PUT $ MACHINUNIT(I,"TONS")      " ton " ;
  PUT XM.M(I):12:0 ;
) ;

PUT // "Tractor size: ", TRACTPOWER:5:1, " kW" ;

PUT // "Number of tractors: ", ZT.L:1:0 ;

PUT // "Fraction of operations performed in various weeks:" ;
PUT / "Numbers in %, though -- = 0% and XX = 100%" ;
PUT // ".      |---- Week numbers ---->" ;
PUT / "Operation " ;
LOOP (K $ SUM(J, JXK(J,K)),
  PUT WEEKNUMBER(K):3:0 ;
) ;
LOOP(J,
  PUT / J.TL:11 ;
  LOOP(K $ SUM(JJ, JXK(JJ,K)),
    IF (OPERPLAN(J,K) EQ 0,
      PUT " --" ;
    ELSE IF (OPERPLAN(J,K) LT 100,
      PUT OPERPLAN(J,K):3:0 ;
    ELSE
      PUT " XX"
    ) ; ) ;
) ;

PUT // "Effective capacities for various operations:" ;
PUT // "Operation      Capacity Unit" ;
LOOP(J,
  PUT / J.TL:12 ;
  PUT $ (XO.L(J) LT 1000) XO.L(J):9:1, " ton/hour" ;
  PUT $ (XO.L(J) GE 1000) HAPERHOUR(J):9:1, " ha/hour" ;
) ;

PUT /// "MISCELLANEOUS DERIVED RESULTS" ;

PUT // "Working profile (man-hours):"
PUT // " Week          Used      + Rest(*) = Available | Percentage" ;
PUT / "Number      Hours ( % )      Hours ( % )      Hours ( % ) | utilisation(**)" ;
PUT / "-----" ;
LOOP(K $ SUM(J, JXK(J,K)),
  PUT / WEEKNUMBER(K):6:0 ;

```

```

PUT WRKPROFILE(K):10:1, " (", WRKPRF_PCT(K):3:0, ")" ;
PUT WRKREST(K):8:1, " (", WRKRST_PCT(K):3:0, ")" ;
PUT T(K):8:1, " (", T_PCT(K):3:0, ")" |" ;
PUT WRKWTUSED(K):9:0 ;
) ;
PUT / "-----" ;
PUT / "Total":6, WRKPRF_SUM:10:1, WRKRST_SUM:14:1, T_SUM:14:1 ;
PUT / "-----" ;
PUT / " (*) 'Rest' includes man-hours which coincide with periods" ;
PUT / "      when workability does not allow field operations." ;
PUT / " (**) Utilisation of man-hours which coincide with periods" ;
PUT / "      when workability allows field operations." ;

PUT /// "Percentage utilisation of machine- and tractor hours during" ;
PUT / "periods when workability allows field operations:" ;
PUT / "Numbers in %, though -- = 0% and XX = 100%" ;

PUT // ".      |---- Week numbers ---->" ;
PUT / "Machine      " ;
LOOP (K $ SUM(J, JXK(J,K)),
  PUT WEEKNUMBER(K):3:0 ;
) ;
LOOP(I,
  PUT / I.TL:11 ;
  LOOP(K $ SUM(JJ, JXK(JJ,K)),
    IF (MACH_PCT(K,I) EQ 0,
      PUT " --" ;
    ELSE IF (MACH_PCT(K,I) LT 100,
      PUT MACH_PCT(K,I):3:0 ;
    ELSE
      PUT " XX"
    ) ; ) ;
) ;
PUT / "Tractor(s)" ;
LOOP(K $ SUM(JJ, JXK(JJ,K)),
  IF (TRACTO_PCT(K) EQ 0,
    PUT " --" ;
  ELSE IF (TRACTO_PCT(K) LT 100,
    PUT TRACTO_PCT(K):3:0 ;
  ELSE
    PUT " XX"
  ) ; ) ;
) ;

PUTCLOSE RES ;

```

Appendix D
Include files for the GAMS model

The data for the GAMS model have been isolated in separate input data files which are included in the GAMS model through INCLUDE statements during execution (see the program listing in Appendix C). There are 12 such include files, which all have the filename extension .INC:

CAPFAC .INC	MACHDATA .INC	MACHINES .INC
	MACHUNIT .INC	
MANHOUR .INC	MISCDATA .INC	OPERATIO .INC
	OPERDATA .INC	
PERMACH .INC	OPERSEQ .INC	OPERTYPE .INC
	OPERWEEK .INC	

All files are ASCII files. Below, the structure of each data file is described, and simple examples of the contents of the files are shown. The examples are all based on the same very simple example case.

D.1 MACHINES.INC

The file MACHINES .INC defines unique names of the machinery types which should be included and optimised in the model (the set of names correspond to the index i defined in Section 2.2). The names used should agree with the GAMS naming conventions for elements. An element name has to start with a letter (A, ..., Z) or a digit (0, ..., 9) followed by letters, digits, plus (+), hyphen (-) or underscore (_) in any order. The length is limited to 10 characters (Brooke *et al.*, 1992).

The structure of the data file is as follows:

<i>MachineName</i>
<i>MachineName</i>
<i>MachineName</i>
:

The ellipses (:) mean that more names may follow. A very simple example of the file MA-CHINES . INC might be:

```
PLOUGH  
HARROW  
SOWINGMACH  
COMBINE  
TRAILER
```

D.2 OPERATIO.INC

The file OPERATIO . INC defines unique names of the operations to be performed (the set of names corresponds to the index j defined in Section 2.2). The names used should agree with the GAMS naming conventions for elements. An element name has to start with a letter (A, ..., Z) or a digit (0, ..., 9) followed by letters, digits, plus (+), hyphen (-) or underscore (_) in any order. The length is limited to 10 characters (Brooke *et al.*, 1992).

The structure of the data file is as follows:

```
OperationName  
OperationName  
OperationName  
:
```

The ellipses (:) mean that more names may follow. A very simple example of the file OPERATIO . INC might be:

```
PLOUGHING  
HARROWING1  
HARROWING2  
SOWING  
HARVEST
```

D.3 MACHUNIT.INC

The file MACHUNIT . INC defines the size units used for the machinery types defined in the file MACHINES . INC (see Table 4 in Appendix A). The structure of file is as follows:

MachineName.Unit
MachineName.Unit
MachineName.Unit
:

Here, *Unit* is either METRE, TONSPRHOURL or TONS (i.e. m, t/h or t). A simple example of the file MACHUNIT . INC might be:

PLOUGH .	METRE
HARROW .	METRE
SOWINGMACH .	METRE
COMBINE .	TONSPRHOURL
TRAILER .	TONS

D.4 MACHDATA.INC

The file MACHDATA . INC contains data for each of the machines defined in the file MACHINES . INC. The content of the file should be structured in a table as follows:

	FIO	FII	THETA	XMMIN	XMMAX
<i>MachineName</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>
<i>MachineName</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>
<i>MachineName</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>
:	:	:	:	:	:

Here, *r.r* denotes a numeric value (real) entered in a way which obeys the syntax rules given by Brooke *et al.* (1992) (a style similar to that in most other computer languages). The column headings of the table corresponds to quantities defined in Chapter 2:

$FIO = \varphi_{0,i}$ $FII = \varphi_{1,i}$ $THETA = \theta_i$ $XMMIN = x_i^{M,min}$ $XMMAX = x_i^{M,max}$

The units of these quantities are as follows: FIO is in DKK, FI1 is in DKK/*SizeUnit*, THETA is in W/*SizeUnit*, while XMMIN and XMMAX are in *SizeUnit*, where *SizeUnit* stands for the size unit used for the machinery type concerned (i.e. m, t/h or t as defined in the file MACHUNIT . INC).

The file MACHDATA . INC might look like this:

	FIO	FI1	THETA	XMMIN	XMMAX
PLOUGH	597	4080	41700	0.80	1.60
HARROW	-4620	1155	10000	5.00	9.00
SOWINGMACH	-4270	2688	6000	2.00	8.00
COMBINE	-36194	30904	0	2.30	7.63
TRAILER	730	1089	10000	3.40	18.16

D.5 OPERDATA.INC

The content of the file OPERDATA . INC is a number of data values for each of the operations defined in the file OPERATIO . INC. The data must be structured as follows:

	ALPHA	BETA	GAMMA	DELTA	TOPT	A	U	R	Q	W
OperationName	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>i</i>	<i>r.r</i>
OperationName	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>i</i>	<i>r.r</i>
OperationName	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>r.r</i>	<i>r.r</i>	<i>i</i>	<i>i</i>	<i>r.r</i>
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Here *r.r* and *i* denote numeric values (real and integer, respectively) entered in accordance with the GAMS syntax rules. The column headings of the table corresponds to quantities defined in Chapter 2 and Appendix A:

ALPHA = α_j

BETA = β_j

GAMMA = γ_j

DELTA = δ_j

TOPT = t_j^{opt}

A = A_j

U = U_j

R = r_j

Q = q_j

W = w_j

The units of these quantities are as follows: ALPHA is in DKK, BETA is in *EffCapUnit*-DKK, GAMMA is in *EffCapUnit*-DKK/W, DELTA is in DKK/week, TOPT is a week number, A is in m², U is in t/m² if *EffCapUnit* = t/h and dimensionless if *EffCapUnit* = m²/h, while R, Q and W are dimensionless where *EffCapUnit* means the effective capacity associated with the operation concerned (m²/h or t/h, see Appendix A).

An example of a OPERDATA . INC file might be as follows:

	ALPHA	BETA	GAMMA	DELTA	TOPT	A	U	R	Q	W
PLOUGHING	197.2	2.31E6	8.8	0	12	22000	1.0	1	1	0.65
HARROWING1	32.6	1.66E6	8.8	0	12	22000	1.0	1	1	0.75
HARROWING2	32.6	1.66E6	8.8	0	12	22000	1.0	1	1	0.75
SOWING	57.7	1.70E6	8.8	321.6	12	22000	1.0	1	1	0.75
HARVEST	774.3	1.09E3	0.0039	160.8	33	22000	0.0004	2	1	0.65

D.6 OPERMACH.INC

The file OPERMACH . INC contains information about which machines are used in each operation (corresponding to the sets M_j ($j = 1, \dots, N^O$) defined in Section 2.5). The structure of the file is as follows:

<i>OperationName.MachineNames</i>
<i>OperationName.MachineNames</i>
<i>OperationName.MachineNames</i>
<i>⋮</i>

where *MachineNames* indicates a single machine or a set of machines. In case of one machine *MachineNames* is simply the name of that machine (one of the machine names listed in the file MACHINES . INC). In case of a set of machines *MachineNames* is a list of machine names enclosed in parenthesis and separated by commas: (*MachineName*, *MachineName*, ...). The file might look like this:

PLOUGHING	. PLOUGH
HARROWING1	. HARROW
HARROWING2	. HARROW
SOWING	. SOWINGMACH
HARVEST	. (COMBINE, TRAILER)

D.7 OPERSEQ.INC

The information stored in the file OPERSEQ . INC ensures that the operations will be performed in proper sequence (corresponds to the set F_j ($j = 1, \dots, N^O$) defined in Section 2.5). The information is structured as follows:

```
OperationName.OperationName
OperationName.OperationName
OperationName.OperationName
:
```

Each line in the file tells which one of two operations must be performed first: the operation on the right hand side of the dot “.” has to be performed before the operation on the left hand side. An example of the content of this file is shown below:

```
HARROWING1 . PLOUGHING
HARROWING2 . HARROWING1
SOWING . HARROWING2
HARVEST . SOWING
```

D.8 OPERTYPE.INC

The file OPERTYPE . INC should tell whether the machines involved in the performance of an operation are operating simultaneously or by turns (corresponding to the parameter h_j ($j = 1, \dots, N^0$) defined in Section 2.5). The file is structured like this:

```
OperationName.OperationType
OperationName.OperationType
OperationName.OperationType
:
```

where *OperationType* is PARALLEL if the machines are operating simultaneously and SERIAL if the machines are operating in turns. If a given operation does only involve one machine, there will be no difference between specifying PARALLEL or SERIAL. Therefore, the specification of *OperationType* for such operations may be omitted.

An example of the file OPERTYPE . INC might be as follows:

```
PLOUGHING . SERIAL
HARVEST . PARALLEL
```

D.9 OPERWEEK.INC

The file OPERWEEK . INC contains information for each operation about the maximum time span within which it is realistic to place the performance of the operation concerned. The time spans are specified as week numbers. The data in OPERWEEK . INC corresponds to the parameters t_j^{\min} and t_j^{\max} ($j = 1, \dots, N^O$) defined in Section 2.5 and should be structured as follows:

OperationName.WeekNumbers
OperationName.WeekNumbers
OperationName.WeekNumbers
:

where *WeekNumbers* denotes one or more weeks: a single week, *nn*, is indicated by *Wnn*, and a span of weeks from week *nn* to week *mm* is indicated by (*Wnn*Wmm*) . The week numbers *nn* and *mm* should be chosen among 01, 02, ..., 52.

The file OPERWEEK . INC might look like this:

PLOUGHING . (W12*W15)
HARROWING1 . (W12*W15)
HARROWING2 . (W12*W15)
SOWING . (W12*W15)
HARVEST . (W32*W34)

D.10 MANHOUR.INC

The file MANHOUR . INC contains number of expected man-hours available for field work in each week (corresponding to the parameter T_k ($k = 1, \dots, 52$) defined in Section 2.5). The data are structured as follows:

W01 *r.r*
W02 *r.r*
W03 *r.r*
:
W52 *r.r*

where *r.r* stands for man-hours. An example of the file MANHOUR . INC is:

W01	89.6
W02	55
W03	77.3
:	:
W52	65.4

where “:” indicates that the number of available man-hours for weeks which are not shown should also be specified in the real file.

D.11 CAPFAC.INC

In the file CAPFAC . INC factors which are used for conversion of the sizes of machines to their effective capacities in connection with different operations are stored (corresponding to the parameter s_{ji} ($j = 1, \dots, N^O$; $i = 1, \dots, N^M$) defined in Section 2.5 and Appendix A). The file must specify conversion factors for all the combinations of operations and machines which are listed in the file OPERMACH . INC. The form of the file is as follows:

<i>OperationName.MachineName</i>	<i>r.r</i>
<i>OperationName.MachineName</i>	<i>r.r</i>
<i>OperationName.MachineName</i>	<i>r.r</i>
:	

where *r.r* indicates a conversion factor (real number). The unit of a conversion factor is hours per metre if the size of the corresponding *MachineName* is measured in metres, dimensionless if the size is measured in tonnes per hour, and hours if the size is measured in tonnes (see Appendix A).

PLOUGHING . PLOUGH	0.0001384
HARROWING1 . HARROW	0.0001384
HARROWING2 . HARROW	0.0001384
SOWING . SOWINGMACH	0.0001587
HARVEST . COMBINE	1.176
HARVEST . TRAILER	0.20

D.12 MISCDATA.INC

The contents of the data file MISCDATA . INC has the following form:

```
CT = r.r ;  
PT = r.r ;  
TW = r.r ;
```

where $r.r$ represents a real number. The names CT, PT and TW correspond to the quantities c^T , p^T and T^W defined in Sections 2.4 and 2.5. CT (dimensionless) is the fixed cost fraction for tractors, PT is the purchase price per power unit when buying a tractor (DKK/W), and TW is the number of working hours during one week. The contents of the file might look like this:

```
CT = 0.14 ;  
PT = 5.24 ;  
TW = 70 ;
```

Appendix E

Example of output produced by the GAMS model

The GAMS model produces two files containing output data. The first one is the GAMS default output file which contains extensive information. However, this file contains much more information than needed and in a form which is not easily comprehensible. Most of the information in the file is intended for model checking. Therefore, the GAMS model has been extended with a section which produces another output file, `RESULT.TXT`, which presents the optimisation results and the derived results in a more comprehensible way.

The following pages shows an example of the output data in the file `RESULT.TXT`. The output data in this example correspond to optimisation run 2, described in Chapter 3. Note that the names of machines are abbreviations which meet the 10 character limitation in GAMS. The names of the operations take the form *Opr_n_Fieldid*, where *Opr* is an abbreviation of the type of operation, *Fieldid* is a number identifying the field where the operation takes place, and *n* is a serial number counting the number of times this type of operation has been performed in the field concerned. This naming convention should merely be regarded as an example. In general, the names for machines and operations are optional. The only requirement is that the identifiers should be unique, following the GAMS rules for labels (see Appendix D, Section D.1 and D.2 or Booke *et al.*, 1992).


```

=====
| RESULTS FROM SOLVING NON-LINEAR MODEL FOR |
| OPTIMISATION OF THE SIZE OF FARM MACHINERY |
=====

```

Model status (see 'GAMS, A User's Guide'): Locally optimal

Solver status (see 'GAMS, A User's Guide'): Normal completion

VALUE OF OBJECTIVE FUNCTION

```

Total annual costs:      516645 DKK
of this
  fixed costs:           308293 DKK
  operating costs:       183604 DKK
  timeliness costs:      24748 DKK

```

VALUES OF DECISION VARIABLES

Size of machines:

Machine	Size	Interval	Unit	Marg.price (DKK)
PLOUGH	1.5	0.8 - 1.6	metre	0
LSTHARROW	7.1	3.4 - 7.8	metre	0
SBEDCULT	6.2	5.0 - 9.0	metre	0
STUBBLCULT	2.1	2.0 - 7.9	metre	0
SOWMACH	3.8	2.0 - 8.0	metre	0
TROMLE	7.1	1.8 - 12.2	metre	0
SPRAYER	10.0	10.0 - 24.0	metre	597
FERTAPPL	2.4	2.4 - 24.0	ton	1020
MOWER	3.2	1.7 - 3.2	metre	-4057
EXACTCHOP	21.1	16.5 - 28.3	ton/hour	0
TIPTRAILER	3.9	3.4 - 18.2	ton	0
FORHARVEST	16.0	16.0 - 35.0	ton/hour	38
BEETHARVST	33.0	33.0 - 73.0	ton/hour	625
UNLDWAGON1	5.9	4.5 - 12.0	ton	0
UNLDWAGON2	5.9	4.5 - 12.0	ton	0
SLURRYTANK	6.4	6.0 - 29.0	ton	0
STRAWBALER	5.9	5.9 - 8.1	ton/hour	1890
BALETRAIL1	2.5	0.4 - 2.5	ton	-21883
BALETRAIL2	0.4	0.4 - 2.5	ton	1777
COMBINE	2.3	2.3 - 7.6	ton/hour	10690
PRECSEEDDR	2.0	2.0 - 8.1	metre	2405

Tractor size: 63.6 kW

Number of tractors: 3

Fraction of operations performed in various weeks:

Numbers in %, though -- = 0% and XX = 100%

[illegible]

[illegible]

[illegible]

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Effective capacities for various operations:

Operation	Capacity	Unit
PLOU_1_5	0.9	ha/hour
PLOU_1_51	0.9	ha/hour
PLOU_1_7	0.9	ha/hour
PLOU_1_2	0.9	ha/hour
HARR_1_5	4.5	ha/hour
HARR_1_51	4.5	ha/hour
HARR_2_5	4.5	ha/hour
HARR_2_51	4.5	ha/hour
HARR_3_5	5.1	ha/hour
HARR_3_51	5.1	ha/hour
HARR_1_7	4.5	ha/hour
HARR_2_7	4.5	ha/hour
HARR_3_7	5.1	ha/hour
HARR_1_2	4.5	ha/hour
HARR_2_2	4.5	ha/hour
HARR_3_2	5.1	ha/hour
HARR_1_4	4.5	ha/hour
HARR_1_41	4.5	ha/hour
HARR_1_8	4.5	ha/hour
HARR_2_4	5.1	ha/hour
HARR_2_41	5.1	ha/hour
HARR_2_8	5.1	ha/hour
HARR_1_9	4.5	ha/hour
HARR_2_9	5.1	ha/hour
HARR_1_1	4.5	ha/hour
HARR_1_11	4.5	ha/hour
HARR_2_1	5.1	ha/hour
HARR_2_11	5.1	ha/hour
HARR_1_3	4.5	ha/hour
HARR_2_3	5.1	ha/hour
HARR_1_31	4.5	ha/hour
HARR_2_31	5.1	ha/hour
HARR_1_91	4.5	ha/hour
HARR_2_91	5.1	ha/hour
SOW_1_1	2.4	ha/hour
SOW_1_3	2.4	ha/hour
SOW_1_11	2.4	ha/hour
SOW_1_31	2.4	ha/hour
SOW_1_91	2.4	ha/hour
SOW_1_2	2.4	ha/hour
SOW_1_4	2.4	ha/hour
SOW_1_41	2.4	ha/hour
SOW_1_8	2.4	ha/hour
SOW_1_9	2.4	ha/hour
SOW_1_5	1.3	ha/hour
SOW_1_51	1.3	ha/hour
SOW_1_7	1.3	ha/hour
ROLL_1_71	4.5	ha/hour
ROLL_1_9	4.5	ha/hour

ROLL_1_11	4.5 ha/hour
ROLL_1_31	4.5 ha/hour
ROLL_1_3	4.5 ha/hour
ROLL_1_2	4.5 ha/hour
ROLL_1_4	4.5 ha/hour
ROLL_1_41	4.5 ha/hour
ROLL_1_8	4.5 ha/hour
ROLL_1_32	4.5 ha/hour
ROLL_1_10	4.5 ha/hour
ROLL_1_1	4.5 ha/hour
ROLL_1_5	4.5 ha/hour
ROLL_1_51	4.5 ha/hour
ROLL_1_7	4.5 ha/hour
MANUR_1_11	12.9 ton/hour
SLUR_1_11	25.5 ton/hour
SLUR_1_2	21.9 ton/hour
SLUR_1_4	24.5 ton/hour
SLUR_1_41	23.6 ton/hour
SLUR_1_8	19.3 ton/hour
SLUR_1_1	28.9 ton/hour
SLUR_1_9	14.5 ton/hour
SLUR_1_7	19.3 ton/hour
SLUR_1_5	7.0 ton/hour
SLUR_1_51	7.1 ton/hour
FERT_1_6	5.8 ton/hour
FERT_1_32	5.8 ton/hour
FERT_1_3	5.8 ton/hour
FERT_1_71	5.8 ton/hour
FERT_1_8	5.8 ton/hour
FERT_1_5	5.8 ton/hour
FERT_1_51	5.8 ton/hour
FERT_1_7	5.8 ton/hour
FERT_2_6	5.8 ton/hour
FERT_2_32	5.8 ton/hour
FERT_2_71	5.8 ton/hour
SPRAY_1_32	5.6 ha/hour
SPRAY_1_10	5.6 ha/hour
SPRAY_1_6	5.6 ha/hour
SPRAY_1_3	5.6 ha/hour
SPRAY_2_32	7.2 ha/hour
SPRAY_2_10	7.2 ha/hour
SPRAY_2_6	7.2 ha/hour
SPRAY_1_1	7.2 ha/hour
SPRAY_1_11	7.2 ha/hour
SPRAY_1_31	7.2 ha/hour
SPRAY_1_91	7.2 ha/hour
SPRAY_1_2	7.2 ha/hour
SPRAY_1_4	7.2 ha/hour
SPRAY_1_41	7.2 ha/hour
SPRAY_1_8	7.2 ha/hour
SPRAY_1_9	7.2 ha/hour
SPRAY_2_41	7.2 ha/hour
SPRAY_1_5	7.2 ha/hour
SPRAY_1_51	7.2 ha/hour

SPRAY_2_5	7.2 ha/hour
SPRAY_2_51	7.2 ha/hour
SPRAY_1_7	7.2 ha/hour
SPRAY_2_7	7.2 ha/hour
SPRAY_3_5	7.2 ha/hour
SPRAY_3_51	7.2 ha/hour
SPRAY_3_7	7.2 ha/hour
SPRAY_3_6	7.2 ha/hour
PLOU_1_9	1.1 ha/hour
PLOU_1_3	1.1 ha/hour
PLOU_1_91	1.1 ha/hour
PLOU_1_10	1.1 ha/hour
PLOU_2_5	1.1 ha/hour
PLOU_2_51	1.1 ha/hour
PLOU_2_7	1.1 ha/hour
PLOU_1_32	1.0 ha/hour
PLOU_2_2	1.0 ha/hour
PLOU_1_31	1.0 ha/hour
PLOU_1_4	1.0 ha/hour
PLOU_1_41	1.0 ha/hour
PLOU_1_8	1.0 ha/hour
STUBC_1_6	1.3 ha/hour
STUBC_1_10	1.3 ha/hour
STUBC_2_6	1.3 ha/hour
STUBC_3_6	1.3 ha/hour
STUBC_4_6	1.3 ha/hour
HARR_3_9	3.9 ha/hour
HARR_4_9	4.5 ha/hour
HARR_3_3	4.0 ha/hour
HARR_4_3	4.5 ha/hour
HARR_3_91	4.0 ha/hour
HARR_4_91	4.5 ha/hour
HARR_1_10	4.0 ha/hour
HARR_2_10	4.5 ha/hour
SOW_2_9	2.5 ha/hour
SOW_2_3	2.5 ha/hour
SOW_2_91	2.5 ha/hour
SOW_1_10	2.5 ha/hour
FERT_2_3	5.8 ton/hour
FERT_3_3	5.8 ton/hour
FERT_2_5	5.8 ton/hour
FERT_2_51	5.8 ton/hour
FERT_2_7	5.8 ton/hour
FERT_1_1	5.8 ton/hour
FERT_1_2	5.8 ton/hour
FERT_1_4	5.8 ton/hour
FERT_1_41	5.8 ton/hour
FERT_2_8	5.8 ton/hour
FERT_3_71	5.8 ton/hour
FERT_1_31	5.8 ton/hour
FERT_1_11	5.8 ton/hour
FERT_2_2	5.8 ton/hour
FERT_4_3	5.8 ton/hour
FERT_2_4	5.8 ton/hour

FERT_2_41	5.8 ton/hour
FERT_3_8	5.8 ton/hour
SPRAY_3_32	5.6 ha/hour
SPRAY_2_91	5.6 ha/hour
SPRAY_2_1	5.6 ha/hour
SLUR_2_9	14.5 ton/hour
SLUR_1_3	21.9 ton/hour
SLUR_1_91	7.2 ton/hour
SLUR_1_10	7.2 ton/hour
SLUR_1_6	5.4 ton/hour
SLUR_1_32	26.5 ton/hour
SLUR_2_2	21.9 ton/hour
SLUR_1_31	22.7 ton/hour
SLUR_2_4	24.5 ton/hour
SLUR_2_8	19.3 ton/hour
GHARV_1_31	1.8 ton/hour
GHARV_1_6	1.8 ton/hour
GHARV_1_10	1.8 ton/hour
GHARV_1_11	1.8 ton/hour
GHARV_1_91	2.0 ton/hour
GHARV_1_9	2.0 ton/hour
BALE_1_31	2.1 ton/hour
BALE_1_6	2.1 ton/hour
BALE_1_10	2.1 ton/hour
BALE_1_11	2.1 ton/hour
BALE_1_91	2.1 ton/hour
SCUT_1_9	13.6 ton/hour
TOP_1_5	13.6 ton/hour
TOP_1_51	13.6 ton/hour
TOP_1_7	13.6 ton/hour
BHARV_1_5	28.0 ton/hour
BHARV_1_51	28.0 ton/hour
BHARV_1_7	25.9 ton/hour
CHOP_1_1	17.9 ton/hour
MOW_1_2	1.8 ha/hour
MOW_1_4	1.8 ha/hour
MOW_1_41	1.8 ha/hour
MOW_1_8	1.8 ha/hour
CHOP_1_2	16.9 ton/hour
CHOP_1_4	16.9 ton/hour
CHOP_1_41	16.9 ton/hour
CHOP_1_8	16.9 ton/hour
MOW_1_3	2.3 ha/hour
MOW_2_3	2.3 ha/hour
CHOP_1_3	17.9 ton/hour
CHOP_2_3	17.9 ton/hour
MOW_1_1	2.3 ha/hour
CHOP_2_1	17.9 ton/hour
MOW_2_2	2.3 ha/hour
MOW_2_4	2.3 ha/hour
MOW_2_41	2.3 ha/hour
MOW_2_8	2.3 ha/hour
CHOP_2_2	17.9 ton/hour
CHOP_2_4	17.9 ton/hour

CHOP_2_41

17.9 ton/hour

CHOP_2_8

17.9 ton/hour

MISCELLANEOUS DERIVED RESULTS

Working profile (man-hours):

Week Number	Used Hours (%)	+ Rest(*) Hours (%)	= Available Hours (%)	Percentage utilisation(**)
12	54.1 (82)	12.0 (18)	66.1 (100)	100
13	67.7 (82)	14.5 (18)	82.2 (100)	100
14	56.7 (86)	9.4 (14)	66.1 (100)	100
15	48.5 (59)	33.7 (41)	82.2 (100)	81
16	0.0 (0)	66.1 (100)	66.1 (100)	0
17	3.4 (11)	28.2 (89)	31.6 (100)	21
18	5.5 (35)	10.0 (65)	15.5 (100)	71
19	3.0 (10)	28.6 (90)	31.6 (100)	18
20	0.9 (6)	14.7 (94)	15.5 (100)	11
21	0.0 (0)	82.2 (100)	82.2 (100)	0
22	1.1 (2)	65.1 (98)	66.1 (100)	3
23	12.6 (15)	69.6 (85)	82.2 (100)	26
24	0.0 (0)	66.1 (100)	66.1 (100)	0
27	8.6 (10)	73.7 (90)	82.2 (100)	16
28	42.9 (65)	23.2 (35)	66.1 (100)	100
29	53.4 (65)	28.8 (35)	82.2 (100)	100
30	43.0 (65)	23.1 (35)	66.1 (100)	100
31	116.1 (65)	62.7 (35)	178.8 (100)	100
32	22.1 (14)	140.6 (86)	162.7 (100)	21
33	99.9 (56)	78.9 (44)	178.8 (100)	86
34	1.4 (1)	161.3 (99)	162.7 (100)	1
35	15.1 (19)	62.5 (81)	77.6 (100)	35
36	1.4 (2)	60.1 (98)	61.5 (100)	4
37	0.0 (0)	77.6 (100)	77.6 (100)	0
38	29.1 (47)	32.4 (53)	61.5 (100)	86
39	17.5 (10)	161.3 (90)	178.8 (100)	16
40	11.1 (7)	151.7 (93)	162.7 (100)	13
41	65.5 (37)	113.3 (63)	178.8 (100)	69
42	30.8 (50)	30.8 (50)	61.5 (100)	100
43	38.8 (50)	38.8 (50)	77.6 (100)	100
44	30.8 (50)	30.8 (50)	61.5 (100)	100
45	39.8 (51)	37.9 (49)	77.6 (100)	100
46	30.8 (50)	30.8 (50)	61.5 (100)	100
47	40.8 (53)	36.8 (47)	77.6 (100)	100
48	30.8 (50)	30.8 (50)	61.5 (100)	100
49	15.5 (20)	62.1 (80)	77.6 (100)	40
Total:	1038.6	2050.3	3088.9	

(*) 'Rest' includes man-hours which coincide with periods when workability does not allow field operations.

(**) Utilisation of man-hours which coincide with periods when workability allows field operations.

	----- Week numbers ----->																																																
Machine	12	13	14	15	16	17	18	19	20	21	22	23	24	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49													
PLOUGH	10	24	17	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	12	--	4	--	--	--	--	--	46	--	57	30														
LSTHARROW	8	6	--	9	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	3	--	1	--	--	--	--	--	--	--	--	--														
SBEDCULT	11	8	--	18	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	4	--	1	--	--	--	--	--	--	--	--	--														
STUBBLCULT	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	3	3	3	3	--	4	--	--	--	--	--	13	--	28	0	0														
SOWMACH	17	13	--	5	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	8	--	--	--	--	--	--	--	--	--	--	--														
TROMLE	--	2	--	27	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
SPRAYER	1	--	2	6	--	8	16	7	2	--	3	10	--	--	--	--	--	--	--	--	--	--	--	1	8	--	--	--	--	--	--	--	--	--	--														
FERTAPPL	2	3	2	3	--	2	--	2	--	--	1	--	4	2	0	--	4	--	3	--	1	--	--	--	--	--	3	--	--	--	--	--	--	--	--														
MOWER	--	--	--	--	--	--	--	--	--	--	--	3	--	3	--	13	--	14	--	--	7	--	--	--	9	--	19	--	--	--	--	--	--	--	--														
EXACTCHOP	--	--	--	--	--	--	--	--	--	--	--	6	--	4	31	7	31	41	--	--	9	--	--	--	8	--	26	--	--	--	--	--	--	--	--														
TIPTRAILER	2	3	2	3	--	2	--	2	--	--	1	--	4	2	19	--	29	10	51	--	1	--	--	--	--	--	3	--	--	--	--	--	--	--	--														
FORHARVEST	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	12	--	--	--	1	--	--	--	--														
BEETHARVST	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	21	29	37	29	13	--	--	--	--														
UNLDWAGON1	--	0	22	--	--	--	--	--	--	--	--	6	--	4	31	7	31	41	--	--	9	--	--	--	8	--	47	29	37	29	13	--	--	--	--														
UNLDWAGON2	--	--	--	--	--	--	--	--	--	--	--	6	--	4	31	7	31	41	--	--	9	--	--	--	8	--	47	29	37	29	13	--	--	--	--														
SLURRYTANK	46	60	51	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	44	--	24	--	--	--	--	57	42	83	31	14														
STRAWBALER	--	--	--	--	--	--	--	--	--	--	--	--	--	--	12	--	16	7	29	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
BALETRAIL1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	12	--	16	7	29	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
BALETRAIL2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
COMBINE	--	--	--	--	--	--	--	--	--	--	--	--	--	--	18	--	25	10	48	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
PRECSEEDDR	--	--	--	26	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--														
Tractor(s)	32	40	39	33	--	4	5	3	1	--	1	10	--	8	32	25	31	68	8	39	1	13	1	--	25	14	10	60	29	37	29	37	29	37	29	15													

Appendix F

List of symbols and notations

The following list explains the meaning of the mathematical symbols and notations used in this report (exclusive appendices). The units associated with the symbols are given in square brackets, if relevant.

F.1 Symbols

- A or A_j is the field area to be treated (in operation j) [m^2].
- b is the theoretical working width [m].
- B_b is the expenses for fuel and oil per operating hour and per unit of theoretical working width [$\text{DKK}/(\text{m h})$].
- B_k is the expenses for fuel and oil per operating hour and per unit of theoretical harvesting capacity, K [DKK/t].
- B_m is the expenses for fuel and oil per operating hour and per unit load capacity [$\text{DKK}/(\text{t h})$].
- c_i is fixed annual costs of machine i expressed as a fraction of its purchase price [dimensionless].
- C_v is the operating cost [DKK].
- d is a coefficient which should be multiplied by the tractor power to give the expected repair and maintenance costs of the tractor per working hour [$\text{DKK}/(\text{W h})$].
- e is the field efficiency [dimensionless].
- $f()$ is a cost function [DKK].
- F_j is the set of operations which must precede operation j .
- h_j is an indicator variable which indicates if the machines used in operation j are operating simultaneously ($h_j = 0$) or by turns ($h_j = 1$).
- i is an index for numbering of machines in the farm machinery system ($i = 1, \dots, N^M$).
- j is an index for numbering of the operations to be performed during the year ($j = 1, \dots, N^O$).
- K is the theoretical harvesting capacity [t/h].

- k is an index for numbering the weeks during the year ($k = 1, \dots, 52$).
- L is the labour cost [DKK/h].
- M is the mass to be transported [t].
- m is the load capacity [t].
- M_j is the set of machines used in operation j .
- N^M is the total number of machines (exclusive tractors).
- N^O is the total number of operations.
- P is the tractor power [W].
- p_{b0} is the constant term to be used when calculating the purchase price of a machine as a function of its theoretical operating width (price = $p_{b1}b + p_{b0}$) [DKK].
- p_{b1} is the slope coefficient to be used when calculating the purchase price of a machine as a function of its theoretical operating width (price = $p_{b1}b + p_{b0}$) [DKK/m].
- p_{k0} is the constant term to be used when calculating the purchase price of a harvesting machine as a function of its theoretical harvesting capacity (price = $p_{k1}K + p_{k0}$) [DKK].
- p_{k1} is the slope coefficient to be used when calculating the purchase price of a harvesting machine as a function of its theoretical harvesting capacity (price = $p_{k1}K + p_{k0}$) [DKK/(t/h)].
- p_{m0} is the constant term to be used when calculating the purchase price of a machine as a function of its load capacity (price = $p_{m1}m + p_{m0}$) [DKK].
- p_{m1} is the slope coefficient to be used when calculating the purchase price of a machine as a function of its load capacity (price = $p_{m1}m + p_{m0}$) [DKK/t].
- P_i^M is the purchase price of machine i [DKK].
- $p_{0,i}^M$ is the constant of the linear relationship between the purchase price, P_i^M , and size, x_i^M , of machine i ($P_i^M = p_{1,i}^M x_i^M + p_{0,i}^M$) [DKK].
- $p_{1,i}^M$ is the slope of the linear relationship between the purchase price, P_i^M , and size, x_i^M , of machine i ($P_i^M = p_{1,i}^M x_i^M + p_{0,i}^M$) [DKK/m, DKK/(t/h) or DKK/t].
- P^T is the purchase price of a tractor [DKK].
- p^T is the proportional factor of the relationship between the purchase price, P^T , and the size (power) of a tractor ($P^T = p^T x^T$) [DKK/W].

- q_j is the number of tractors used in operation j .
- r is the expected repair and maintenance costs of a machine per working hour, expressed as a fraction of the purchase price [h^{-1}].
- r_j is the number of workers involved full-time in operation j .
- $s_{j,i}$ is a proportionality constant which determines the relation between the size of machine i and its effective capacity in operation j (size = $s \times$ effective capacity) [h/m , dimensionless or h].
- T_k is the expected number of man-hours available for field work during week k [h].
- T^w is the number of working hours during one week [h].
- t_j^{opt} is the optimum week for the performance of operation j (as far as optimisation of crop return is concerned) [dimensionless].
- U is the crop yield [t/m^2].
- U_j is defined depending on the unit of x_j^O . If the unit of x_j^O is t/h , which is the case in operations where material is applied to or removed from the field (e.g. application of slurry or grain harvest), U_j is simply the applied or removed amount per unit of area (t/m^2). If the unit of x_j^O is m^2/h , U_j is 1 (e.g. harrowing and ploughing).
- v is the driving speed in the field [m/h].
- w_j is the fraction of the working hours which is left for the performance of operation j when the expected hours with unfavourable weather, soil or crop conditions have been left out [dimensionless].
- $x_{j,k}$ is the fraction of the j 'th operation being performed in the k 'th week [dimensionless].
- \underline{x}^M is the size of the i 'th machine. The size of a machine is either measured as theoretical working width (in metres, e.g. for harrows and ploughs), theoretical harvesting capacity (in tonnes per hour, e.g. for combines and exact choppers) or load capacity (in tonnes, e.g. for trailers).
- $x_i^{M,\text{max}}$ is the maximum size of machine i [m , t/h or t].
- $x_i^{M,\text{min}}$ is the minimum size of machine i [m , t/h or t].
- x_j^O is the effective field capacity of the machine or set of machines used for the performance of the j 'th operation. The measuring unit is either m^2/s or t/h , depending on the size unit(s) associated with the machine (or set of machines) used for the operation in question.

- κ^T is the power of the tractors in the farm machinery system [W].
- λ^T is the number of tractors in the farm machinery system.

F.2 Greek symbols

- α_j is a parameter related to the fuel, repair and maintenance costs of the machinery used for operation j (defined in Appendix A) [DKK].
- β_j is a parameter related to labour costs and repair/maintenance costs of the machinery used for operation j (defined in Appendix A) [(DKK m²)/h or [DKK t)/h].
- γ_j is a parameter related to repair and maintenance costs of the tractor(s) used in operation j (defined in Appendix A) [(DKK m²)/(h W) or (DKK t)/(h W)].
- δ_j is a timeliness factor related to operation j (defined in Appendix A) [DKK/week].
- θ_i is the required tractor power per size unit of machine i [W/m, W/(t/h) or W/t].
- ι is an alternative index for numbering of machines in the farm machinery system ($\iota = 1, \dots, N^M$).
- κ is an alternative index for numbering of the weeks during the year ($\kappa = 1, \dots, 52$).
- τ is the time used for transportation of one load [h].
- $\varphi_{0,i}$ is defined as $\varphi_{0,i} = c_i^M p_{0,i}^M$ [DKK].
- $\varphi_{1,i}$ is defined as $\varphi_{1,i} = c_i^M p_{1,i}^M$ [DKK/m, DKK/(t/h) or DKK/t].
- ψ is defined as $\psi = c^T p^T$ [DKK/W].

F.3 Notations

The notations are given in terms of examples.

- $\min_x f(x)$ find the value of x , which minimises the function $f(x)$.
- $j | i \in M_j$ the indices j where the index i is a member of the index set M_j .
- \forall for all (e.g. $\forall j$, which means for all values of the index j).
- g^Y a variable name. It does not mean g raised to the Y 'th power. Y is to be considered as a part of the variable name.

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