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Estimation of air-temperature in a barley canopy by means of statistical methods

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Ph.D. dissertation



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Preface

The present report was prepared as part of my Ph.D. programme at The Royal Veterinary and Agricultural University (RVAU), Copenhagen.

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Ege Friis January 1992

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Summary

In the present work, simple statistical models for predicting air temperature in a barley canopy on the basis of macro-meteorological and phenological observations were developed and validated.

In the CROPm models, meteorological predictor variables are "above crop" variables and phenological variables refer to the crop as a whole. In the CANOPY models, phenological variables refer to some part of the canopy and meteorological variables are "in canopy" variables.

In the linear CROPm models the meteorological predictor variables were air temperature at 2.0 m, net radiation above canopy and wind speed at the top of the canopy. The phenological variables were height and crop area index of the crop.

The predictor variables included in the non-linear CANOPY models were a term expressing the change in net radiation at the relevant level in the canopy and a term expressing the wind speed at that level. The values of the predictor variables were calculated under the assumption that the density function of the vertical distribution of the crop area index followed a "half-circle" function.

Models were developed for four levels within the crop: 0.05, 0.15, 0.30 and 0.50 m above ground. For all levels, a CROPm model type was found to have the smallest residual standard deviation: 1.44, 1.28, 1.22 and 1.04 °C, respectively. The residuals of these models were strongly autocorrelated, and accounting for this by means of an autoregressive process reduced the standard deviation to about 0.65 °C for all levels. The resicual standard deviations of the CANOPY models for the different levels were in the range 1.53 to 1.83 °C.

The "best" non-autoregressive linear CROPm model type was validated using data sets for two years from another location. The data sets included data on air temperature in a barley crop at 0.2 m above the ground. The prediction errors of the model in these validations were 1.45 and 1.79 $^{\circ}$ C, respectively. For operational applications, models having a prediction error above 1 $^{\circ}$ C would probably not be of much value.

Danish summary

I nærværende undersøgelse er der foretaget udvikling og validering af simple statistiske modeller til beregning af luft-temperaturen i en bygafgrøde på grundlag af makro-meteorologiske og fænologiske observationer.

I CROPm modellerne anvendes "over afgrøde" meteorologiske variable og de fænologiske variable referer til afgrøden som helhed. I CANOPY modellerne refererer fænologiske variable til et udsnit af afgrøden, og der anvendes "i afgrøde" meteorologiske variable.

I de lineære CROPm modeller indgår følgende meteorologiske størrelser som forklarende variable: lufttemperatur i 2.0 m, nettostråling over afgrøden og vindhastighed lige over afgrøden. De fænologiske, forklarende variable er afgrødens højde og (blad-)areal indeks.

I de ikke-lineære CANOPY modeller indgår to forklarende variable. En variabel beskriver ændringen i nettostrålingen i en given højde i afgrøden, og en anden variabel beskriver vindhastigheden i den pågældende højde. Værdier af de forklarende variable blev beregnet under antagelse af, at tæthedsfunktionen for den vertikale fordeling af (blad-)areal indeks kan beskrives med en "halvcirkel" funktion.

Der er udviklet modeller for fire niveauer i afgrøden: 0.05, 0.15, 0.30 og 0.50 m over jordoverfladen. For alle niveauer findes, at en CROPm modeltype har de mindste standardafvigelser på residualerne, henholdsvis 1.44, 1.28, 1.22 og 1.04 °C. Residualerne i disse modeller er stærkt autokorrelerede. Såfremt der tages hensyn til dette gennem en autoregressiv proces, reduceres standardafvigelserne til omkring 0.65 °C for alle niveauer. Standardafvigelserne på residualerne for CANOPY modellerne for de fire niveauer ligger i intervallet: 1.53 til 1.83 °C.

Den "bedste" ikke-autoregressive CROPm model type er valideret ved anvendelse af to års data fra en anden lokalitet. Disse datasæt omfatter registreringer af lufttemperaturen i en bygafgrøde i højden 0.2 m over jordoverfladen. Modellens prediktionsfejl i disse valideringer er henholdsvis 1.45 og 1.79 °C. Modellerne med en prediktionsfejl større end 1 °C vil formodentlig ikke være af større værdi i operationelle anvendelser.

1 Introduction

A wide range of processes in the soil-plant-atmosphere system are regulated by micrometeorological conditions rather than the macrometeorological conditions. Consequently, when modelling these processes one might expect that models based on micrometeorological input variables would improve model performance.

Operational applications of models based on micrometeorological input variables are somewhat restricted as micrometeorological observations are not available in real time. In Denmark crop relevant micrometeorological observations are only carried out at a few places and solely for research purposes. In contrast, macrometeorological observations are available for a large number of stations (Mikkelsen, 1991).

One way to make these models operational would be to estimate micrometeorological variables by means of models using ordinary macrometeorological observations and relevant crop observations as input.

The plant-soil microclimate is a highly complex and dynamic system. Modelling this system on sound bio-physical principles involves rather sophisticated models as those described by Goudriaan (1977), Meyers and Paw U (1987) and Wu (1990). In general, reliable microclimate simulation models are input demanding (in terms of constants as well as driving variables), involve extensive computation and work on a small time scale. In effect, the operational applications of this type of models are restricted.

The present study was undertaken to evaluate the potential of simple, empirical, statistical models for estimating microclimatic variables. The study focuses on models for estimating air temperature in a barley canopy on the basis of macrometeorological variables and macro-properties of the canopy. To the knowledge of the author a similar approach has not been described in the literature.

2 Data materials

The present study was based on meteorological and biological data from two sources: a) study on micrometeorology of spring barley, Research Centre Foulum and b) the micrometeorological field station at Dept. of Agrometeorology, Research Centre Foulum.

2.1 Study on micrometeorology of spring barley (Formyre data)

The experiment was carried out in 1989 at Research Centre Foulum. The experimental design and procedures are described in detail in Friis (1991).

The experimental area was located in the Formyre mark (~ 20 ha) which was grown with spring barley cv. Inge. The experimental area totalled 1 ha (100 × 100 m) and was divided into three plots grown with spring barley cv. Digger. The plant density of the plots was approximately 200, 400 or 800 plants \cdot m⁻², respectively.

Meteorological and biological variables used in this work were observed in the plot with a plant density of approximately 400 plants $\cdot m^{-2}$, which is equivalent to the density normally found under farm conditions. Agronomic data for the barley plot are given in Table 2.1.

Location	Foulum mi	Formyre	
Year	1988	1989	1989
Cultivar	Sewa	Sewa	Digger
Fertilizer			
NPK, kg · ha ⁻¹	115-22-55	115 - 22 - 55	115 - 22 - 55
Plant density,			
plants \cdot m ⁻²	293	338	389
Sowing	19.04.89	05.04.89	17.04.89
Emergence	06.05.89	18.04.89	27.04.89
Harvest	18.08.89	21.08.89	25.08.89

Table 2.1: Agronomic data for barley plots at Formyre and Foulum.

The variables used were:

a. air temperature at 0.05, 0.15, 0.30 and 0.50 m above the ground.

- b. air temperature at 2.0 m above the ground.
- c. net-radiation measured 1 m above the actual height of the canopy.
- d. wind velocity at 2.0 m above the ground.
- e. global radiation at 2.0 m above the ground.
- f. height of canopy.
- g. leaf area index (LAI), crop area index (CAI), "dead-leaf" area index (DAI).

Air temperature and global radiation at 2.0 m above the ground were measured just outside the plot.

Wind speed at 2.0 m was not observed directly but estimated by interpolation of profile measurements of wind speed. The interpolation procedure is described below.

Meteorological variables were recorded each minute and stored as 10 minute averages. Air temperatures at 0.05, 0.15, 0.30 and 0.50 m above the ground were observed at two sites simultaneously. The distance between the sites was approximately 6 m. Corresponding observations at the two sites — in terms of time and height — are taken as replicates in this study. Other meteorological variables were observed at one site only.

In this study 1 hour averages were calculated on the basis of the 10 minute values. In case three or more 10 minute values were missing, the hourly value was set to missing.

Air temperatures were measured in a modified Aanderaa-type radiation screen (Foulum screen) (Aanderaa, 1986). Later comparative measurements of air temperature in the Foulum screen and in an ordinary Stevenson screen showed that measurements in the Foulum screen were subjected to radiation errors (Fig. 2.1). The magnitude of the error showed some dependence on wind velocity and net radiation.

Although the radiation errors were quite large under certain conditions, no attempt was made to correct the temperature observations. The reason for this was that a correction function based on net radiation and wind velocity would not be directly applicable to temperature measurements in the canopy, as neither net radiation nor wind velocity in the canopy was recorded. Both variables might be estimated using proper models, but such an approach was considered outside the scope of the present study.

Differences between replicate observations of air temperature in the canopy showed diurnal and in some cases more frequent fluctuations. An example is given i Fig. 2.2, which shows differences between replicate observations at levels 0.05 and 0.50 m above ground. At both levels the largest absolute differences were observed in the daytime. During the period shown in Fig. 2.2 the largest daily difference at the 0.50 m level increased and changed sign. Summary statistics for differences between replicate measurements are shown in Table 2.2.



Figure 2.1: Corresponding temperature observations in Foulum screen and Stevenson screen at 2.0 m. Data for the period 13.07.90 - 06.08.90. X-axis shows wind velocity at 2.0 m. Y-axis shows difference between temperature observations (Foulum minus Stevenson screen). Symbols $a, b, c \ldots g$ refer to net radiation Rn at 1.5 m above ground (short grass): $a: -100 < Rn \le 0$; $b: 0 < Rn \le 100$; $\ldots g: 500 < Rn \le 600$ (all values in Wm⁻²).



Figure 2.2: Temperature difference between replicate measurements of air temperature in the canopy at levels 0.05 and 0.50 m above ground (Formyre data).

Week no.						
start date	Height		Minimum	Maximum	Mean	S
end date	cm	N		°C		
23	05	78	-0.45	1.14	0.22	0.37
	15	78	-1.51	1.59	-0.06	0.55
05.06.89	30	78	-0.08	0.25	0.03	0.05
11.06.89	50	78	-0.12	0.12	0.01	0.05
24	05	146	-0.42	3.72	1.18	1.04
	15	146	-1.36	2.56	0.47	0.73
12.06.89	30	146	-3.98	2.24	0.14	0.84
18.06.89	50	146	-0.28	0.43	0.02	0.13
25	05	168	-0.10	3.08	0.97	0.95
	15	168	-0.34	3.17	0.54	0.69
19.06.89	30	168	-3.10	0.69	-0.15	0.46
25.06.89	50	168	-0.43	0.70	0.08	0.21
26	05	168	-0.31	1.83	0.49	0.56
	15	168	-0.35	1.13	0.18	0.27
26.06.89	30	168	-0.65	0.67	-0.03	0.22
02.07.89	50	168	-0.88	0.78	-0.03	0.25
27	05	167	-0.29	2.17	0.55	0.60
	15	167	-0.58	1.16	0.10	0.36
03.07.89	30	167	-0.89	1.08	0.01	0.32
09.07.89	50	107	-1.64	1.31	-0.19	0.55
28	05	156	-0.23	1.64	0.34	0.40
	15	156	-0.24	1.01	0.16	0.26
10.07.89	30	156	-0.97	0.22	-0.11	0.17
16.07.89	50	0			•	•
29	05	168	-0.21	2.09	0.55	0.56
	15	168	-0.28	1.00	0.16	0.26
17.07.89	30	168	-0.71	0.73	-0.02	0.21
23.07.89	50	63	-1.50	0.15	-0.28	0.28
30	05	160	-0.37	2.04	0.44	0.67
	15	160	-0.22	0.83	0.15	0.25
24.07.89	30	160	-0.59	0.44	-0.04	0.20
30.07.89	50	160	-1.06	0.49	-0.21	0.21
31	05	168	-0.73	1.16	0.01	0.36
	15	168	-0.31	0.58	-0.02	0.15
31.07.89	30	168	-1.79	0.36	-0.30	0.44
06.08.89	50	168	-0.86	0.61	-0.13	0.20

Table 2.2: Summary statistics for differences between replicate measurements of air temperature in Formyre barley plot. N is the number of observations, s the standard deviation.

The differences between replicate temperature measurements were probably caused by differences in canopy structure (height, leaf area etc.) in combination with the radiation error associated with the Foulum screen as discussed above. For the reasons mentioned above, no attempt was made to correct the observed temperatures.

The wind profile included 4 anemometers mounted westwards of the mast at heights shown in Table 2.3. In each setup the lowermost anemometer was initially mounted approximately 0.50 m above the top of the canopy.

Table 2.3: Mounting heights (cm) above ground of anemometers on profile mast in the Formyre barley plot.

Level above	Period				
ground	14.06.89 - 23.06.89	23.06.89-10.08.89			
1	85	110			
2	120	145			
3	181	206			
4	285	310			

In estimating wind velocity at 2.0 m, it was assumed that the observed wind-profile followed the standard logarithmic wind-profile equation (Monteith and Unsworth, 1990)

$$u_z = \frac{u_*}{k} \cdot \ln \frac{(z-d)}{z_o} \tag{2.1}$$

- u_z horizontal windspeed at height z, ms⁻¹
- u_* friction velocity, ms⁻¹
- k von Karman's constant, (0.41)
- z height above ground, m
- d zero plane displacement, m
- z_o roughness length, m

d is an equivalent height for absorption of momentum, and $(d + z_o)$ is an equivalent height for zero windspeed. Both d and z_o is dependent on crop structure and wind speed.

In principle d and z_o could be estimated graphically by plotting u against ln(z - d) for various values of d. An iterative procedure in a computer program could also be used.

In the present study a simpler approach was followed as d was estimated using the relation given by Monteith (1973) based on work by Stanhill (1969).

$$d = 0.63 \cdot z_c \tag{2.2}$$

It was assumed that d was constant within a 24 hour period, and d was calculated using observed or linearly interpolated daily values of z_c . Based on the linear relation between u and ln(z-d) in (2.1), wind speed at 2.0 m was then estimated by simple linear interpolation using the relation

$$u_{2.0} = u_1 + \frac{(u_2 - u_1)}{\ln(z_2 - d) - \ln(z_1 - d)} \cdot [\ln(2.0 - d) - \ln(z_1 - d)]$$
(2.3)

 $u_{2.0}$ horizontal wind speed at 2.0 m, ms⁻¹

 u_1 observed horizontal wind speed at the level closest below 2.0 m, ms⁻¹

 u_2 observed horizontal wind speed at the level closest above 2.0 m, ms⁻¹

 z_1 height above ground for u_1 observations, m

 z_2 height above ground for u_2 observations, m

d zero plane displacement, m

The height of the canopy was measured twice weekly in six replicates. From emergence to complete flowering the height of the canopy was taken as the distance from the ground to the top of the canopy. Complete flowering is equivalent to growth stage 69 on the Zadoks scale (Zadoks et al., 1974). In the milk development stage, and later stages the height of the canopy was taken as the distance from the ground to the base of the ear. In this study the height of the canopy was taken as the average of the replicates.

Daily values of canopy height were estimated using simple linear interpolation of observed heights. The interpolation started at the day of emergence and ended at the day of the last observation. At the day of emergence the canopy height was set to zero. The interpolation approach implied that canopy height was taken to be constant within a 24-hour period. Of course, this might not be the case, but for the present study such an assumption was considered an acceptable approximation.

Leaf area index (LAI), crop area index (CAI) and "dead-leaf" area index (DAI) was determined once weekly in two replicates according to the procedure described briefly below. A detailed description af the procedure is given by Plauborg (1990). In the field a representative sample of above ground plant material within a ground area of 0.25 m^2 ($0.48 \times 0.52 \text{ m}$) was cut off as close to the ground as practically possible (2-5 cm above ground level). In the laboratory the weight of the field sample was determined and a sub-sample of known weight was fractioned into *a*) green stems, *b*) green leaves, *c*) green ears and *d*) dead material. Plant elements with a necrotic area of more than 50 % of the total area (determined visually) went into fraction *d* (dead material). Each fraction was weighed, and the total area of the projection of the individual plant elements on a plane was determined using a Licor LI 3100 Area Meter. Based on the measured areas and weights, LAI was calculated for the field sample as the ratio of the area of fractions *a-c* to sample ground area. CAI was calculated as the ratio of the area of all fractions *a-d* to sample ground area. DAI vas calculated as the ratio of the area of fraction *d* to sample ground area.

LAI, CAI and DAI values used in this work were averages of the two replicates.

Daily values of LAI, CAI and DAI were estimated using simple linear interpolation of observed values. The interpolation started at the day of emergence (when LAI, CAI and DAI were set to zero) and ended at the day of the last observation. As for canopy height, an interpolation approach is based on the assumption that the variables were constant within a 24-hour period. For the present study this was considered an acceptable approximation.

2.2 Foulum micrometeorological field station (Foulum data)

The micrometeorological field station at Department of Agrometeorology is located at Research Center Foulum. The facilities and observation program at the station were described by Olesen (1987).

The station is divided into a reference area and four crop plots. The reference area is grown with a lawn-type grass vegetation, which is kept short by frequent cuttings. In the plots, spring barley, winter wheat, perennial ryegrass and peas or rape are grown in rotation.

In this study meteorological and biological observations from the reference area and the barley plot for the years 1988 and 1989 were used. For the barley plot all observations were from the non-irrigated part of the plot. Agronomic data for the barley plots are shown in Table 2.1.

The variables used were:

- a. air temperature at 0.20 m above ground in the barley plot.
- b. air temperature at 2.0 m above ground in the reference area, measured in a Stevenson screen.
- c. net-radiation measured at 1.50 m above ground in the reference area.
- d. wind speed at 2.0 m above ground in the reference area.
- e. height of the barley canopy.
- f. leaf area index (LAI), crop area index (CAI) and "dead-leaf" area index (DAI) of the barley canopy.

Temperatures were observed and stored every 10 minutes. Net radiation was recorded each minute and stored as 10 minute averages.

Hourly means of meteorological variables were calculated on the basis of the 10 minute values. In case three or more 10 minute values were missing, the hourly value was set to missing.

Air temperature in the barley canopy was measured in a type 2773 Aanderaa radiation screen (Aanderaa, 1986). Results of Mortensen and Jensen (1987) indicate that this type of radiation screen does not exclude radiation errors in temperature measurements. In the present work, possible radiation effects on measurements have not been analysed and consequently no corrections have been made.

In the presentation of the general meteorological conditions at Foulum, daily data retrieved from Department of Agrometeorology databases was used. All variables were observed according to standard meteorological procedures.

The height of the barley canopy was in general measured weekly in 1988 and bi-weekly in 1989. However, in both years the observation frequency differed during the season. In this study the height was taken as the mean of eight replicates. Daily values of canopy height were estimated by means of simple linear interpolation according to the procedure described in section 2.1. In 1989 the last record of canopy height was taken at 10.07.89. For the period from 10.07.89 until harvest the canopy height was assumed constant and equivalent to the height at 10.07.89.

LAI, CAI and DAI of the barley canopy were in general determined weekly in 1988 (two replicates) and 1989 (one replicate) according to the procedure described in section 2.1. In 1988 the area of dead material (fraction d) was determined on three occasions only – on dates 20.05.88. 30.05.88 and 04.07.88. However, the weight of fraction d was recorded for all samples at every sampling date. Based on the area-weight ratio on date 04.07.88 the area of dead material was estimated for sampling dates with missing values. In 1989 LAI, CAI and DAI were not determined in the period 19.05.89 to 14.06.89.

The data used for 1988 were means of the two replicates. Daily values of LAI, CAI and DAI were estimated by means of linear interpolation according to the procedure described in section 2.1.

3 Methods

The first section of this chapter gives a brief introduction to the processes regulating air temperature in the canopy. Later sections describe the models and the statistical methods that were used in this study.

3.1 Introduction to processes regulating temperature in a canopy

Basically the temperature of an air-parcel within a canopy is regulated by the energy balance and the physical characteristics of the parcel.

In practice, calculations of energy balance terms and air temperature are facilitated by employment of a canopy model, that specifies relevant characteristics. In the most simple canopy model, the canopy consists of a uniform mono-layer. For this model the energy balance may be written as in 3.1 when assuming that the horizontal net-flow of energy as well as energy terms related to freezing/melting of water and photosynthesis are negligible.

$$Rn + G + C + \lambda E + S = 0 \tag{3.1}$$

Rn net radiation, Wm⁻²

- G soil heat flux, Wm^{-2}
- C sensible heat flux, Wm^{-2}
- λE latent heat flux, Wm^{-2} (evapotranspiration)
- S a storage term, Wm^{-2}

Fluxes directed towards the canopy are taken as positive and fluxes away from the canopy as negative.

The storage term S refers to energy stored in canopy elements (leaves, stems etc.) and the air. According to this simple model the storage term is responsible for changes in air temperature in the canopy. In general S is numerically much smaller that the other terms in the energy balance.

The simple uniform mono-layer canopy model is rather unrealistic for real canopies. As examples, the vertical distribution of biomass or CAI as well as the temperature profile are generally non-uniform. An alternative model is the multi-layer canopy model. In this model the canopy is divided into a number of discrete uniform layers, thus providing a framework for handling vertical variation in biomass, CAI, energy balance terms etc.

Further theoretical and computational details regarding calculation of canopy air temperature using the multi-layer model may be found in Goudriaan (1977).

3.2 Models

In this study canopy air temperature was modeled using two types of empirical models. The model types are termed CROP and CANOPY and may be written in the general form

$$T_{c,z} = T_{2,0} + f(Rn', u', z'_{c}, xAI')$$
(3.2)

 $\begin{array}{ll} T_{c,z} & \text{canopy air temperature at height } z \text{ above ground.} \\ T_{2.0} & \text{air temperature at } 2.0 \text{ m.} \\ f(\ldots) & \text{a function.} \\ Rn' & \text{a net radiation term.} \\ u' & \text{a wind speed term.} \\ z'_c & \text{a crop height term.} \\ xAI' & \text{a LAI or CAI term.} \end{array}$

As can be seen from (3.2) the only input variable related to the energy balance is the net radiation term Rn'.

The terms CROP and CANOPY indicate the sort of variables used as input to the model. In CROP-models phenological variables refer to the crop as a whole and meteorological variables are "above-crop" variables. In CANOPY models phenological variables refer to some part of the canopy and meteorological variables are "in-canopy" variables.

CROP and CANOPY models are formulated on the basis of the hypothesis that i) the numerical value and the sign of the difference $T_{c,z} - T_{2.0}$ depend primarily on Rn' and ii) the numerical value of this difference is reduced with increasing values of u', z'_c and xAI'.

In the CROP models the effects of Rn', u', z'_c and xAI' are considered additive. When analyzing the full data set, condition ii) of the hypothesis requires that the u', z'_c and xAI'terms include a "sign conversion factor", which is dependent on Rn'. In the CANOPY models the effects of Rn', z'_c and xAI' are incorporated into a single term, which is multiplied with an exponential term of u'.

3.2.1 CROP-models

The CROP-models take the form

$$T_{c,z} = T_{2,0} + \beta_1 Rn + \beta_2 \Omega u_c + \beta_3 \Omega z_c + \beta_4 \Omega x A I$$
(3.3)

Rn	net radiation above the crop, Wm^{-2}
u_c	wind speed at crop height z_c , ms ⁻¹
z _c	crop height, m
xAI	leaf area index (LAI) or crop area index (CAI)
$\beta_1, \beta_2, \beta_3, \beta_4$	parameters.
Ω	sign conversion factor.

In some analyses, the full data set (D_z) for $T_{c,z}$ was analyzed with a single model. In these cases the following values were assigned to Ω :

 $\Omega = -1.0$ for observations with $Rn \ge 0$ Wm⁻² $\Omega = +1.0$ for observations with Rn < 0 Wm⁻²

In other analyses, the full data set D_z for $T_{c,z}$ was divided into two subsets depending on the value of Rn. Subset D_z^+ includes observations with $Rn \ge 0$ Wm⁻², and subset D_z^- includes observations with Rn < 0 Wm⁻². When analyzing these data sets, Ω was assigned a value of +1.0.

The wind speed u_c is calculated from the logarithmic wind profile equation

$$u_{c} = \frac{u_{*}}{k} \cdot ln \frac{(z_{c} - d)}{z_{0}}$$
(3.4)

with

$$d = 0.63 \cdot z_c \tag{3.5}$$

$$z_0 = 0.1 \cdot z_c \tag{3.6}$$

$$u_{\star} = \frac{(u_{2.0} \cdot k)}{\ln \frac{(z_u - d)}{z_0}}$$
(3.7)

$$k = 0.41 \tag{3.8}$$

$$z_u = 2.0m \tag{3.9}$$

In the equations above k is von Karman's constant and z_u is height above ground of wind speed observation. Equations (3.5) and (3.6) (Monteith and Unsworth, 1990) are approximations, which are considered to be acceptable in the present context.

3.2.2 CANOPY-models

The CANOPY-models take the form

$$T_{c,z} = T_{2,0} + \beta_1 \left[\frac{dRn_z}{dz} \right]^* \exp(\beta_2 u_z)$$
(3.10)

 Rn_z net radiation at height z in the canopy, Wm⁻²

 u_z wind speed at height z in the canopy

 $\left[\frac{dRn_2}{dz}\right]^*$ average of $\frac{dRn_2}{dz}$ over air layer with thickness equal to the length of temperature sensor (0.05 m). The average is based on data calculated for 0.01 m sub-layers.

 β_1, β_2 parameters.

 Rn_z is estimated as

$$Rn_z = Rn \cdot \exp(-k_{Rn}CAI_{z'}) \tag{3.11}$$

 $CAI_{z'}$ accumulated crop area index between heights z and z_c

 k_{Rn} extinction-coefficient for net radiation in the canopy

According to the literature k_{Rn} is about 0.8 in the nighttime when global radiation is zero (Goudriaan, 1977) and about 0.5 under a clear sky at noon in the summertime (Denmead, 1976; Ross, 1975).

The extinction coefficient k_{Rn} depends on solar elevation and the fractions of diffuse and direct radiation in the global radiation S_i . As a first approximation, this relation was expressed in terms of S_i , and k_{Rn} was assumed to depend linearly on S_i

$$k_{Rn(t)} = 0.8 - 3.6 \cdot 10^{-4} \cdot \sum_{t_1=t-3}^{t} \left(\frac{S_{i(t_1)}}{4}\right)$$
(3.12)

where suffix (t) and (t_1) refers to time in hours. The coefficient to the S_i term was estimated on the basis of Formyre data on S_i under the assumption that k_{Rn} was equal to 0.5 for the maximum value of the S_i term. The approximation (3.12) has not been validated.

The accumulated crop area index between heights z_c and z, $CAI_{z'}$, is expressed as

$$CAI_{z'} = D(z') \cdot CAI \tag{3.13}$$

where D(z') is the distribution function for the vertical distribution of CAI. z' is the distance from the top of the canopy to the level z in the canopy

$$z' = z_c - z \tag{3.14}$$

The vertical distribution of CAI was assumed to follow a "half circle" distribution. The density function d(z') and the distribution function D(z') for a "half circle" distribution are given as (Appendix B):

$$d(z') = \frac{2}{\pi r^2} \sqrt{r^2 - x^2}$$
(3.15)

$$D(z') = \frac{2}{\pi r^2} \left[\frac{1}{2} \left\{ x \sqrt{r^2 - x^2} + r^2 \cdot \arcsin\left(\frac{x}{|r|}\right) \right\} + \frac{\pi r^2}{4} \right]$$
(3.16)

In (3.15) and (3.16) r and x are given by

$$r = \frac{1}{2} \cdot z_c \tag{3.17}$$

$$x = z - \frac{1}{2}z_c (3.18)$$

 $\frac{dRn_z}{dCAI_{z'}}$ is given by

$$\frac{dRn_z}{dCAI_{z'}} = -k_{Rn} \cdot Rn \cdot \exp(-k_{Rn} \cdot CAI_{z'})$$
(3.19)

Inserting equation (3.13) and (3.16) and evaluating $\frac{dCAI_{z'}}{dz}$ from (3.13) and (3.14) gives (Appendix B)

$$\frac{dRn_z}{dz} = k_{Rn} \cdot d(x) \cdot CAI \cdot Rn \cdot \exp[-k_{Rn} \cdot D(x) \cdot CAI]$$
(3.20)

Wind speed in the canopy u_z was approximated by an expression given by Goudriaan (1977)

$$u_z = u_c \cdot \exp(-k_u(1-\frac{z}{z_c}))$$
 (3.21)

where k_u is an extinction coefficient.

Equation (3.21) is only strictly valid under a set of conditions including neutral atmospheric conditions and uniform distribution of CAI.

 k_u is estimated as

$$k_u = \left(\frac{c_d \cdot CAI \cdot z_c}{2l_m i_\omega}\right)^{\frac{1}{2}} \tag{3.22}$$

where

$$l_m = 2\left(\frac{\omega}{\pi \cdot \frac{CAI}{z_c}}\right)^{\frac{1}{2}} \tag{3.23}$$

.

The symbols are

- c_d drag coefficient of leaves, -
- i_{ω} relative turbulence intensity, -
- ω characteristic width of leaves, m
- l_m mean distance between leaves, m

Based on results in Goudriaan (1977) k_u was calculated with w = 0.01 m, $i_w = 0.5$ and $c_d = 0.2$.

	Data set			
Period	symbol	Remarks		
Full period	D_z	$T_{c,0.05}, T_{c,0.15}, T_{c,0.30}$: 15.06 - 26.07, 1989		
		$T_{c,0.50}$: 21.06 - 17.07, 1989		
15.06 - 25.06, 1989	aD_z	In general, the days were clear;		
		almost no precipitation.		
		Crop height, LAI and CAI increasing.		
27.06 - 02.07, 1989	bD_z	Unstable weather conditions.		
		Precipitation every day.		
		Crop variables are almost constant.		
04.07 - 09.07, 1989	$c\overline{D_z}$	In general the weather was clear;		
		almost no precipitation; temperature increasing.		
		Crop height constant, LAI and CAI decreasing.		
20.07 - 26.07, 1989	dD_z	In general the weather was clear;		
		almost no precipitation; temperature increasing.		
		Crop height, LAI and CAI are almost constant.		
		LAI is close to zero.		

Table 3.1: Data periods used in model development. Formyre data.

3.3 Statistical methods

Estimation of parameters in the CROP- and CANOPY-models was carried out using observations in the 1989-Formyre data set.

For each $T_{c,z}$ variable, the data period included in the analyses was the period when the level of temperature observation was within the canopy. Some of the analyses were carried out using data from selected sub-periods. The characteristics of the data periods are summarized in Table 3.1.

In the analyses $\Delta T_{c,z}$, defined as

$$\Delta T_{c,z} = T_{c,z} - T_{2,0} \tag{3.24}$$

was taken as the dependent variable. The analyses included the models mentioned below. As mentioned above, t refers to time in hours.

CROPm-Ω

$$(\Delta T_{c,z})_t = (\beta_1 R n + \beta_2 \Omega u_c + \beta_3 \Omega z_c + \beta_4 \Omega x A I)_t + \varepsilon_t$$
(3.25)

Models including CAI as predictor variable are labeled CROPm- Ω_C and models including LAI are labeled CROPm- Ω_L

CROPm-R

For observations in data subset D_z^- ($Rn < 0 \text{ Wm}^{-2}$):

$$CROPm - R^{-}: \quad (\Delta T_{c,z})_t = (\beta_1 Rn + \beta_2 u_c + \beta_3 z_c + \beta_4 x AI)_t + \varepsilon_t \tag{3.26}$$

For observations in data subset D_z^+ ($Rn \ge 0 \text{ Wm}^{-2}$):

$$CROPm - R^+: \quad (\Delta T_{c,z})_t = (\beta_1'Rn + \beta_2'u_c + \beta_3'z_c + \beta_4'xAI)_t + \varepsilon_t \tag{3.27}$$

Models including CAI as predictor variable are labeled CROPm- R_C and models including LAI are labeled CROPm- R_L

CROPa

$$(\Delta T_{c,z})_t = MT_t + \omega_t$$

$$\omega_t = \sum_{i=1}^n (\phi_i \cdot \omega_{t-i}) + \varepsilon_t \qquad (3.28)$$

CANOPY

$$(\Delta T_{c,z})_t = \left(\beta_1 \left[\frac{dRn_z}{dz}\right]^* \exp(\beta_2 u_z)\right)_t + \varepsilon_t \tag{3.29}$$

Index t indicates observation time. β_i, β'_i , and ϕ_i are parameters. MT is estimated using either the CROPm- Ω or the CROPm-R models. ω is an autoregressive term and ε is a statistical error. The ε_t terms are independent random variables with zero expectation and constant variance.

Statistical analyses were carried out with procedures in the SAS-software package (SAS Institute Inc., 1988a,b).

Estimation of parameters in the CANOPY-models requires application of non-linear regression techniques. The need for this approach is due to the fact that in the Formyre data set $\Delta T_{c,z}$ and $\left[\frac{dR_{n_z}}{dz}\right]^*$ may have different signs. If this was not the case equation (3.28) could be rearranged to read

$$ln\left(\frac{\Delta T_{c,z}}{\left[\frac{dRn_z}{dz}\right]^*}\right) = ln\beta_1 + \beta_2 u_c \tag{3.30}$$

The parameters in this model could be estimated using linear regression techniques.

Estimation of parameters in the CANOPY models was carried out using the Gauss-Newton iteration method provided by the SAS procedure NLIN. With this method it is necessary to specify first derivatives with respect to the parameters. The method also requires initial estimates of the parameters.

Initial parameters were estimated by application of the model 3.30 to the Formyre data subset for which $\Delta T_{c,z}$ and $\left[\frac{dRn_z}{dz}\right]^*$ had identical signs.

Model validation was carried out using observations in Foulum data sets from 1988 and 1989. As the Formyre data includes observations of canopy air temperature at 0.15 and 0.3 m and the Foulum data only includes observations at 0.2 m it was not possible to validate the models "directly". The procedure applied was to calculate model estimates of temperature at heights 0.15 and 0.3 m, and afterwards estimate temperature at 0.2 m by simple linear interpolation.

4 Descriptive statistics

4.1 General meteorological conditions in the 1988 and 1989 growing seasons

The meteorological conditions in the period April to August in 1988 and 1989 are summarized in Table 4.1.

In 1988 monthly mean temperatures were close to the 1961-90 climatic normals. The monthly precipitation was lower than normal for all months except July. Monthly global radiation was close to normal.

In 1989 monthly mean temperatures were close to the corresponding normals. In April and May monthly precipitation was slightly higher than normal, while precipitation in June, July and August was substantially lower than normal. The global radiation was higher than the normal for every month except August.

In relation to model development and validation the months June and July are of particular interest. Daily values of air temperature (at 2.0 m), precipitation (at 1.5 m) and global radiation at Foulum for the these months in 1988 and 1989 are shown in Figure 4.1a,b.

4.2 Meteorological data

Summary statistics for meteorological data of relevance to model development and validation are shown in Table 4.2. It should be noted that for 1989, Foulum and Formyre statistics are based on data from different periods.

Time plots of $\Delta T_{c,z}$ for the Formyre data used in this study are shown in Fig. 4.2. The data periods included in the analyses were those when the level of temperature observation was within the canopy.

As can be seen in Fig. 4.2, all $\Delta T_{c,z}$ variables except $\Delta T_{c,0.05}$ show diurnal variation with a positive maximum in the daytime and a negative minimum in the nighttime. In general, $\Delta T_{c,0.05}$ is found to oscillate with a period of approximately 12 hours, showing positive

	Temperature, °C		Precipitation, mm			Global radiation, $MJ \cdot m^{-2}$				
	(at 2.0 m)				(at 1.5 m)					
Month/	-		Normal			Normal			Normal	
decade	1988	1989	1961-90	1988	1989	1961-90	1988	1989	1961-90	
April	5.0	$\overline{5.7}$	5.5	22	65	35	380	419	386	
1.	4.1	3.3		0	5		88	113		
2.	6.8	8.4		14	18		128	131		
3.	4.0	5.3		7	15		164	175		
May	11.9	$1\overline{0}.8$	10.5	20	45	45	607	645	537	
1.	10.3	9.2	,	9	4		170	174		
2.	11.6	10.6		0	7		216	197		
3.	13.8	12.5		12	34		221	274		
June	15.3	13.8	14.2	34	22	52	635	676	590	
1.	14.0	9.8		19	14		179	175		
2.	14.8	16.5		0	0		261	276		
3.	17.1	15.1		15	8		195	225		
July	15.4	16.2	15.4	79	39	67	494	613	550	
1.	15.8	17.9		18	2		178	229		
2.	15.7	13.1		32	10		159	177		
3.	14.8	17.5		29	27		157	207		
August	14.6	14.5	15.1	63	28	66	428	425	467	
1.	15.2	14.2		7	11		184	136	:	
2.	14.8	16.2		26	9		110	134		
3.	13.9	13.2		30	8		135	156		

Table 4.1: Mean temperatures, sums of precipitation and global radiation from April to August in 1988 and 1989 at Foulum. Data from Olesen (1989, 1990, 1991).

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Figure 4.1: Daily values of air temperature (at 2 m), precipitation (at 1.5 m) and global radiation at Foulum for the months June and July in 1988 (a) and 1989 (b).

Variable	Year	No.	Missing	Mean	Mini-	Maxi-	Std.
		obs.	obs.		mum	mum	dev.
Location: Foulum	Perio	Period: 01.06-20.07					
Temperature at 2.0 m,	1988	1191	9	15.5	7.7	26.9	3.6
reference area,°C	1989	1200	0	14.5	2.5	30.2	4.9
Wind speed at 2.0 m,	1988	1198	2	3.8	0.5	7.4	1.3
reference area, ms ⁻¹	1989	1200	0	2.9	0.5	7.9	1.4
Net radiation at 1.2 m,	1988	1188	12	124.3	-85.0	597.0	173.9
reference area, Wm ⁻²	1989	1198	2	135.1	-83.0	633.0	189.3
Temperature at 0.20 m,	1988	1190	10	14.2	5.2	32.2	4.7
barley plot, °C	1989	1198	2	15.3	1.0	36.7	6.6
Location: Formyre	Perio	1: 15.06	6-26.07				
Temperature at 2.0 m,							
barley plot, °C	1989	995	13	16.8	5.7	31.9	5.2
Wind speed at 2.0 m,							
barley plot, ms ⁻¹	1989	1008	0	2.6	0.2	7.5	1.3
Net radiation,							
barley plot, Wm ⁻²	1989	995	13	151.9	-91.3	647.9	211.6
Temperature at 0.05 m,							
barley plot, °C	1989	995	13	18.2	8.1	34.4	5.6
Temperature at 0.15 m,							
barley plot, °C	1989	995	13	17.6	4.2	39.0	7.1
Temperature at 0.30 m,							
barley plot, °C	1989	995	13	17.5	3.1	40.4	7.5
Temperature at 0.50 m,							
barley plot, °C	1989	995	13	17.1	3.0	39.6	7.3

Table 4.2: Summary statistics for hourly micrometeorological variables (Foulum and Formyre data).



Figure 4.2: Time plots of $\Delta T_{c,z}$ for the period June 15 to July 30, 1989. Formyre data.



Figure 4.3: Time plot of $\Delta T_{c,0.05}$ and $\Delta T_{c,0.30}$ for the period July 1 to July 5, 1989. Formyre data.

maxima around noon and midnight, and (negative) minima in the late afternoon and in the early morning (Fig. 4.3).

Frequency distributions of $\Delta T_{c,z}$ for the Formyre 1989-data are shown in Fig. 4.4. The distributions show expected characteristics regarding mode-values and frequencies of low and high $\Delta T_{c,z}$ values.

Frequency distributions of $\Delta T_{c,20}$ for the Foulum data sets are shown in Fig. 4.5. The frequency of data with absolute values larger than about 2.0 °C was larger in 1989 than in 1988. It is likely that this result is due to the difference in plant density in the barley plots (Table 2.1).

4.3 Phenological data

The height of the barley canopies in Foulum and Formyre plots are shown in Fig. 4.6. Results for Foulum in 1988 do not give a clear indication of the maximum level although observations have been made quite frequently in this period. The reason for this might be that the observation procedure has not been followed strictly with respect to the phenological stages of the crop.



Figure 4.4: Frequency distributions of $\Delta T_{c,z}$ for the Formyre 1989-data. Data for the period June 15 to July 26, 1989. Observations rounded to the nearest multiple of 1 °C.

LAI and CAI of the canopies in Foulum and Formyre plots are shown in Fig. 4.7a,b. Due to the low plant density in the Foulum plot in 1988, LAI and CAI is lower in this plot than in other plots.



Figure 4.5: Frequency distributions of $\Delta T_{c,20}$ for the Foulum 1988- and 1989-data. Data for the period June 1 to July 20. Observations rounded to the nearest multiple of 1 °C.



Figure 4.6: Height of barley canopy in Foulum and Formyre plots. The lines show interpolated values and the symbols show observed values.





Figure 4.7: LAI (a) and CAI (b) of barley canopy in Foulum and Formyre plots. The lines show interpolated values and the symbols show observed values.

5 Results

5.1 Model development

Using the STEPWISE regression technique, $T_{c,z}$ variables were analysed in relation to a number of meteorological and phenological predictor variables. In addition to these variables, six lag-variables holding lagged values of the meteorological predictors were included.

The analyses of $T_{c,z}$ were carried out on all the data sets listed in Table 3.1, and in all cases the $T_{2.0}$ variable was the first to be included in the analyses. This result indicates that in general, fluctuations in $T_{c,z}$ and $T_{2.0}$ show no phase differences. Based on this result it was decided to proceed the analyses using $\Delta T_{c,z}$, defined as in (3.24), as the independent variable.

The Shapiro-Wilk statistic (W) provided by the SAS procedure UNIVARIATE was applied to test the null hypothesis that the independent $\Delta T_{c,z}$ variables were random samples from normal distributions. By definition W is greater than zero and less than or equal to one. Small values of W indicate departure from normality. The results in Table 5.1 show that for all $\Delta T_{c,z}$ variables the Shapiro-Wilk test rejected the null hypothesis at significance level 0.0001. These results were expected from the frequency distributions shown in Fig. 4.4.

Table 5.1: Test of normality of $\Delta T_{c,z}$ variables. N is the number of observations, W is the Shapiro-Wilk statistic and P (..) is the associated probability for testing the hypothesis that the data come from a normal distribution.

Variable	Ν	Wobs	$P(W < W_{obs})$
$\Delta T_{c,0.05}$	995	0.9749	0.0001
$\Delta T_{c,0.15}$	995	0.9194	< 0.0001
$\Delta T_{c,0.30}$	995	0.9410	< 0.0001
$\Delta T_{c,0.50}$	635	0.9490	< 0.0001

Although the Shapiro-Wilk test rejected the normality null hypothesis for the $\Delta T_{c,z}$ variables, the residuals in the different models will be assumed to come from normal distributions.

Table 5.2: Coefficients of correlation (ρ) between predictor variables in CROPm- Ω models. Significance levels for test of H_0 : $\rho = 0$ are indicated by $* * * \sim P(|\rho| > |\rho_{obs}|) \le 0.001$, $** \sim P(|\rho| > |\rho_{obs}|) \le 0.01$, $* \sim P(|\rho| > |\rho_{obs}|) \le 0.05$. Formyre data set D_z .

Variable	N	Rn	$\Omega \cdot u_c$	$\Omega \cdot z_c$	$\Omega \cdot LAI$
Rn	995	-	-	-	-
$\Omega \cdot u_c$	1008	-0.706***	-	-	-
$\Omega \cdot z_c$	1008	-0.691***	0.894^{***}	-	-
$\Omega \cdot LAI$	1008	-0.608***	0.629^{***}	0.770^{***}	-
Ω · CAI	1008	-0.716***	0.839^{***}	0.954^{***}	0.907***

Table 5.3: Coefficients of correlation (ρ) between predictor variables in CROPm-R models. Significance levels for test of $H_0: \rho = 0$ are indicated as in Table 5.2. Formyre data sets D_z^- and D_z^+ .

Data set	Variable	N	Rn	u_c	z _c	LAI
	Rn	389	-	-	-	-
	u_c	389	0.205***	-	-	-
D_z^-	z _c	389	0.162***	0.203^{***}	-	-
	LAI	389	-0.014	-0.304***	0.115*	-
	CAI	389	0	-0.316***	0.184^{***}	0.969^{***}
	Rn	619	-	-	-	-
	u_c	619	0.238***	-	-	-
D_z^+	z _c	619	-0.119**	0.154^{***}	-	-
	LAI	619	0.140***	-0.196***	0.081*	-
	CAI	619	0.138***	-0.205^{***}	0.159^{***}	0.967^{***}

5.1.1 CROPm models

Coefficients of correlation between predictor variables in the CROPm- Ω models are shown in Table 5.2. The correlations are moderate or strong and in all cases highly significant. These results were expected in consideration of the numerical manipulations involved in calculating the predictor variables.

Coefficients of correlation between predictor variables in the CROPm-R models are shown in Table 5.3. The correlations are weak but in most cases significant. As expected the correlation between the alternative predictor variables LAI and CAI is strong and highly significant.

Table 5.4 shows coefficients of correlation between $\Delta T_{c,z}$ variables and predictor variables in the CROPm- Ω models. The correlation between $\Delta T_{c,z}$ and Rn is quite strong except for $\Delta T_{c,0.05}$, but in all cases the correlation is highly significant. The observed values of $\Delta T_{c,z}$ and Rn are shown in Fig. 5.1.



Figure 5.1: Observed values of $\Delta T_{c,z}$ and Rn. Formyre data set D_z .

Table 5.4: Coefficients of correlation (ρ) between $\Delta T_{c,z}$ and predictor variables in CROPm- Ω models. Significance levels for test of H_0 : $\rho = 0$ are indicated as in Table 5.2. Formyre data set D_z .

Variable	$\Delta T_{c,0.05}$	$\Delta \overline{T_{c,0.15}}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
N	995	995	995	635
Rn	0.391***	0.842***	0.886***	0.869***
Ωu_c	-0.221***	-0.521***	-0.571***	-0.555***
Ωz_c	-0.089**	-0.524***	-0.614^{***}	-0.668***
ΩLAI	-0.019	-0.419***	-0.510***	-0.585^{***}
ΩCAI	-0.079*	-0.539***	-0.629***	-0.645***

Table 5.5: Coefficients of correlation (ρ) between $\Delta T_{c,z}$ and predictor variables in CROPm-R models. Significance levels for test of $H_0: \rho = 0$ are indicated as in Table 5.2. Formyre data sets D_z^- and D_z^+ .

Variable		$\Delta T_{c,0.05}$	$\Delta T_{c,0.15}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
	N	376	376	$37\overline{6}$	236
	Rn	-0.177***	0.253***	0.333***	0.362^{***}
	u_c	-0.186***	0.389^{***}	0.498^{***}	0.562^{***}
D_z^-	z_c	-0.059	0.196^{***}	0.219***	-0.219***
	LAI	-0.037	-0.297***	-0.374***	-0.263***
	CAI	-0.014	-0.231***	-0.305***	-0.295***
	N	619	619	619	399
	Rn	0.489***	0.793***	0.833***	0.812***
	u_c	0.305***	0.114^{**}	0.067	-0.002
D_z^+	z_c	-0.151***	-0.197***	-0.113**	0.168^{***}
	LAI	-0.159***	-0.131**	-0.084*	-0.129
	CAI	-0.16 ***	-0.128**	-0.059	0.013

The plots in Fig. 5.1 show that the relation between $\Delta T_{c,0.05}$ and Rn differs from the relations for other $\Delta T_{c,z}$ variables. For all $\Delta T_{c,z}$ variables the plots indicate that the relation between $\Delta T_{c,z}$ and Rn differs depending on whether Rn is positive or negative. It should be noted that the sign of $\Delta T_{c,z}$ and Rn may be different. This observation conflicts with condition i) of the model hypothesis presented in chapter 3. However, no attempts have been made to analyze this presumption in further detail.

Coefficients of correlation between $\Delta T_{c,z}$ and predictor variables in the CROPm-*R* models are shown in Table 5.5. For data in the D_z^+ data set, correlation coefficients for *Rn* are in general substantially larger than coefficients for other predictor variables. The correlation between $\Delta T_{c,0.05}$ and *Rn* is moderate, while corresponding correlations for other $\Delta T_{c,z}$ variables are quite strong.

For data in D_z^- the largest coefficients are found for predictor variable u_c for all $\Delta T_{c,z}$

Table 5.6: R^2 -values for CROPm- Ω_C and CROPm- R_C models. Formyre data sets. The results for CROPm- R_C models are summary R^2 -values, calculated on the basis of "sum of squares" data for the relevant CROPm- R_C^- and CROPm- R_C^+ models.

		Variable			
Model	Data set	$\Delta T_{c,0.05}$	$\Delta T_{c,0.15}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
$CROPm-\Omega_C$	D_z	0.514	0.701	0.754	0.728
	aD_z	0.649	0.754	0.775	0.731
	bD_z	0.373	0.553	0.715	0.723
	cD_z	0.403	0.699	0.782	0.785
	dD_z	0.753	0.917	0.924	-
CROPm- R_C	D_z	0.574	0.768	0.842	0.838
	aD_z	0.724	0.880	0.902	0.914
	bD_z	0.558	0.668	0.875	0.909
	cD_z	0.445	0.849	0.911	0.937
	dD_z	0.869	0.945	0.954	-

variables. As expected from Fig. 5.1 the sign of the coefficient of correlation between $\Delta T_{c,z}$ and Rn is negative for $\Delta T_{c,0.05}$, but positive for other $\Delta T_{c,z}$ variables. A similar pattern is found for u_c . It was unexpected that coefficients of correlation for u_c were numerically larger than coefficients for Rn. These results may partly be due to the positive correlation between u_c and Rn (Table 5.3).

Model R^2 -values for CROPm- Ω_C and CROPm- R_C models are shown in Table 5.6. The data sets used are described in Table 3.1. For all $\Delta T_{c,z}$ variables and data sets (periods) the R^2 -value of the CROPm- R_C model is larger than the R^2 -value for the CROPm- Ω_C model. For all data sets, CROPm- $R_C R^2$ -values for $\Delta T_{c,0.15}$, $\Delta T_{c,0.30}$, $\Delta T_{c,0.50}$ are considerably larger than R^2 -values for $\Delta T_{c,0.05}$.

In general, R^2 -values for the CROPm- R_C models were smaller for the bD_z dataset than for the other sets aD_z , cD_z and dD_z . As mentioned in Table 3.1 the period covered by bD_z had rain almost every day while the other periods had almost no rain.

The data sets listed in Table 5.6 were also analysed using CROPm- Ω_L and CROPm- R_L "LAI-models". In general, R^2 -values for these models were of the same order of magnitude as the R^2 -values for the "CAI-models" shown in Table 5.6. However, in most cases and in particular for the bD_z dataset, "CAI-models" had slightly larger R^2 -values than "LAImodels".

Statistics and estimates of CROPm- R_C models are shown in Table 5.7. For all $\Delta T_{c,z}$ variables the standard deviation (s) was larger for the CROPm- R_C^+ model than for the CROPm- R_C^- model. The standard deviation of the CROPm- R_C models was in the range 1.04 °C ($\Delta T_{c,0.50}$) to 1.44 °C ($\Delta T_{c,0.05}$).

	1		Variable			
Model	Data set		$\Delta T_{c,0.05}$	$\Delta T_{c,0.15}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
CROPm-R_	D_z^-	R ²	0.609	0.596	0.767	0.825
, i i i i i i i i i i i i i i i i i i i		5	0.94	0.96	1.00	0.94
		$Rn: \beta_1$	-9.67 · 10 ^{-3***}	$11.37 \cdot 10^{-3***}$	16.96 · 10 ⁻³ ***	$15.60 \cdot 10^{-3***}$
	1	$u_c: \beta_2$	-0.216	0.545***	0.820***	1.314***
		$z_c: \beta_3$	1.157**	0.360	0.246	-2.774***
		CAI: β_4	0.062	-0.342***	-0.490***	-0.134
$CROPm-R_C^+$	D_z^+	R^2	0.567	0.799	0.857	0.8425
-		8	1.68	1.45	1.34	1.11
		$Rn: \beta'_1$	5.20 · 10 ^{-3***}	12.24 · 10 ^{-3***}	13.73 · 10 ^{-3***}	11.23 · 10 ^{-3***}
		$u_c: \beta'_2$	0.926***	-0.438***	-1.001***	-1.213***
		$z_c: \beta'_3$	-0.286	1.518***	2.878***	3.732***
		CAI: β'_4	-0.230***	-0.518***	-0.572***	-0.601***
CROPm-R _C	D _z	R^2	0.574	0.768	0.842	0.838
		s	1.44	1.28	1.22	1.04

Table 5.7: Statistics and estimates for CROPm- R_c models. Significance levels for test of $H_0: \beta = 0$ are indicated as in Table 5.2. Formyre data sets D_z , D_z^- and D_z^+ .

The signs of the Rn and u_c parameters are in agreement with the model hypothesis for all $\Delta T_{c,z}$ except $\Delta T_{c,0.05}$. As expected from the $\Delta T_{c,0.05}$ data in Fig. 5.1 the signs of the Rn and u_c parameters are negative in the CROPm- R_c^- model. In the CROPm- R_c^+ models the sign of the u_c parameter for $\Delta T_{c,0.05}$ is positive, while it is negative for the other variables.

The signs of the z_c and CAI parameters do not agree consistently with the model hypothesis.

Residual plots for the CROPm- R_C models are shown in Fig. 5.2. As expected from the results in Table 5.7 the range of the residuals are quite wide for all $\Delta T_{c,z}$ variables. The residual plots show no distinct patterns giving indication of particular weaknesses in the models.

Time plots of the residuals are shown in Fig. 5.3. For all $\Delta T_{c,z}$ variables, but in particular for $\Delta T_{c,0.05}$, the residuals show seasonal trends. Analyses of the meteorological conditions (Fig. 4.1a,b) did not reveal any obvious explanation for these trends. The residuals did not show consistent diurnal variation for any of the $\Delta T_{c,z}$ variables.

5.1.2 CROPa-models

In the analyses presented in this paper, the trends (MT) in the autoregressive CROPa models are equal to the corresponding CROPm- R_C models, and thus ω is equal to the residual of these models. Fig. 5.3 shows time-plots of ω -values for all $\Delta T_{c,z}$ variables. As mentioned above, visual inspection of the time series reveals local trends, which in turn indicate that the series may not be stationary.

Correlograms of the ω variables in the CROPa models are shown in Fig. 5.4. As expected the ω variables are serially correlated in a seasonal (diurnal) manner. For $\Delta T_{c,0.05}$ and $\Delta T_{c,0.50}$



Figure 5.2: Residual plots for CROPm- R_C models. Formyre data set D_z .



Figure 5.3: Time plots of residuals of CROPm- R_C models. Formyre data set D_z .



Figure 5.4: Correlograms of ω in CROPa models. Formyre data set D_z .

Table 5.8: Statistics and estimates for the CROPa models. Significance levels for test of $H_0: \phi = 0$ are indicated as in Table 5.2. AIC is the Akaike Information Criterion. Formyre data set D_z .

		Variable			
Model		$\Delta T_{c,0.05}$	$\Delta T_{c,0.15}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
MT: CROPm- R_C		see Table 5.7			
AR(2) process	R^2	0.794	0.739	0.713	0.634
	AIC	1992.3	1988.5	1986.6	1225.6
	s	0.65	0.66	0.65	0.63
	ϕ_1	-1.130***	-1.029***	-0.993***	-0.919***
	ϕ_2	0.289***	0.211***	0.186***	0.164^{***}
CROPa	R^2	0.912	0.941	0.955	0.941
	s	0.65	0.66	0.65	0.63

the correlogram shows seasonal peaks at lags 10 and 24. For $\Delta T_{c,0.15}$ and $\Delta T_{c,0.30}$ a seasonal peak appears at lag 24.

For all $\Delta T_{c,z}$, the correlograms indicate that ω might follow an auto-regressive (AR) process rather than a moving-average (MA) process, as the auto-correlation functions do not show a "cut-off" point.

Partial correlograms of the ω variables are shown in Fig. 5.5. For an AR-process the partial correlograms should have a distinct "cut-off" point. Although this condition is hardly met in the correlograms in Fig. 5.5, the results indicate that for all $\Delta T_{c,z}$, ω follows approximately an AR(2) process.

Estimation of parameters in the AR process in the CROPa models was carried out on the basis of the CROPm- R_C model residuals, and by means of the SAS procedure AUTOREG according to the Yule-Walker method (SAS, 1988b).

The following AR processes were analysed:

In general, R^2 -values of the AR[1,2,...] processes were about 0.01 larger than the corresponding R^2 values for AR(2) processes. However, referring to the principle of parsimony the AR(2) process was accepted as a satisfactory model for the ω variables.

Statistics and estimates for the CROPa models are shown in Table 5.8. The parameters of the AR(2) processes showed consistency with respect to the sign, and the parameters



Figure 5.5: Partial correlogram of ω in CROPa models. Formyre data sets D_z .



Figure 5.6: Residual plots for CROPa models. Formyre data set D_z .

were in all cases highly significant. For all $\Delta T_{c,z}$ variables the CROPa models fit the data considerably better than the CROPm- R_C models. The standard deviations of the CROPa models are about 0.65 °C, while for the CROPm- R_C models standard deviations are in the range 1.04 to 1.44 °C.

The values of the ϕ -estimates confirm the supposition that the AR series may not be stationary. For a stationary AR(2)process the parameters must fulfil three conditions, one of which is $-\phi_1 + \phi_2 < 1$. This condition is not met for any of the AR(2) processes.

Residual plots for the CROPa models are shown in Fig. 5.6. As expected from Table 5.8, the ranges of the residuals are narrower than for the corresponding CROPm- R_C models (Fig. 5.2). For all $\Delta T_{c,z}$ variables the residuals scatter rather uniformly over the whole range of prediction values.

5.1.3 CANOPY models

Statistics and estimates for the CANOPY models are summarized in Table 5.9. As expected from the plots in Fig. 5.1, the results for $\Delta T_{c,0.05}$ differ from those of other $\Delta T_{c,z}$ variables, as the R^2 value is substantially lower and the sign of the β_2 parameter estimates is reversed. For all $\Delta T_{c,z}$ variables the CANOPY model fits data less well than the corresponding CROPm- R_C model.

Table 5.9: Statistics and estimates of CANOPY models. Significance levels for test of $H_0: \beta = 0$ are indicated as in Table 5.2. Formyre data set D_z .

Variable	$\Delta T_{c,0.05}$	$\Delta T_{c,0.15}$	$\Delta T_{c,0.30}$	$\Delta T_{c,0.50}$
R^2	0.423	0.653	0.636	0.666
s	1.67	1.58	1.83	1.53
β_1 (initial)	$38.7 \cdot 10^{-3}$	$28.8\cdot 10^{-3}$	$21.7\cdot10^{-3}$	$15.0 \cdot 10^{-3}$
β_1 (final)	$9.4 \cdot 10^{-3***}$	$10.0 \cdot 10^{-3***}$	$10.3 \cdot 10^{-3***}$	$12.9 \cdot 10^{-3***}$
β_2 (initial)	-3.43	-3.95	-2.80	-1.68
β_2 (final)	1.88***	-1.23***	-2.48***	-2.6***

Application of CANOPY models to datasets D_z^- and D_z^+ separately improved model performance slightly, but model R^2 values for the full data sets D_z were still substantially lower than the R^2 values for the CROPm- R_C models.

Residual plots for the CANOPY-models are shown in Fig. 5.7. As can be seen the ranges of the residuals are wide and the patterns of the residuals are strongly non-uniform. These features show that the CANOPY-models are inadequate for modelling the air temperature in the canopy.

5.2 Model validation

Estimation of canopy air temperature on the basis of observations at ordinary meteorological stations could be carried out using the CROPm or the CANOPY models. CROPa models are excluded as past observations of the temperature variable in question have to be known.

As the CROPm- R_C models were found to fit data better than the CANOPY models, only CROPm- R_C validation results will be reported.

Results of validations of the CROPm- R_C models on $T_{c,0.20}$ data from Foulum in the years 1988 and 1989 are shown in Fig. 5.8a,b. For the 1988-validation, the mean and the standard deviation of the prediction error are -0.02 °C and 1.45 °C, respectively. For the 1989-validation the corresponding results are 0.46 °C and 1.79 °C, respectively.



Figure 5.7: Residual plots for CANOPY models. Formyre data set D_z .



Figure 5.8: Time plot of prediction errors. Validation of CROPm- R_C models on $T_{c,0.20}$ data from Foulum for the years 1988 (a) and 1989 (b).

The prediction errors show obvious seasonal trends in both the 1988- and the 1989-validation. These trends are to some extent related to the weather conditions and crop characteristics (Fig. 4.1a,b, 4.6 and 4.7). In 1988 the period from June 10 to June 25 is dry and sunny, and crop height and CAI are increasing. The period from June 26 to July 20 is in general somewhat cloudy and rainy with a rather large number of precipitation days. The crop height and CAI is slowly decreasing in this period.

In 1989 the relations are more obscure. As an example, periods of generally positive prediction errors are found to be associated with rainy as well as sunny weather conditions.

The prediction errors appear to exhibit a diurnal variation (Fig. 5.9a,b). In 1989 hourly mean values of the predition errors show a tendency of being positive in the daytime. These results could be due to differences in the net radiation measured at Foulum and Formyre. As can be seen in Fig. 5.10, net radiation observations in the daytime are generally lower at Foulum than at Formyre. This fact might cause increased prediction errors. As pointed out in chapter 2, net radiation is measured above a barley crop at Formyre, but above short grass at Foulum.

In 1988 the hourly mean values of prediction errors show a tendency of being negative in the middle of the day, and slightly positive in the early morning and the late afternoon. The net radiation effect discussed above was also present in 1988, as the measuring equipment and procedure was not changed from 1988 to 1989. In view of this fact, additional factors must be responsible for the trend in the prediction errors. These might probably include the weather conditions in general as well as the sparse canopy in the barley plot at Foulum in 1988 (Fig. 4.7).



Figure 5.9: Hourly mean values of prediction errors. \times symbols indicate mean values. Bars indicate the interval: mean \pm standard deviation of the mean. Validation of CROPm- R_C models on $T_{c,0.20}$ data from Foulum for the years 1988 (a) and 1989 (b).



Figure 5.10: Corresponding hourly values of net radiation measured at Foulum (x-axis) and Formyre (y-axis). Data from the period June 15 to July 26, 1989.

6 Discussion

Among the models analysed in this work, the CROPa models were found to fit the Formyre data better than the CROPm- R_C and the CANOPY models. Indeed the CROPa models fitted all the $\Delta T_{c,z}$ data sets rather well. However, one has to be cautious in taking this as evidence of the models general properties for predicting canopy air temperatures. As pointed out, the residuals of the trend models of the CROPa models (i.e. the CROPm- R_C model) showed evidence of non-stationarity. Furthermore, the residuals were strongly auto-correlated, and this may be a symptom of lack of fit of the trend model (SAS Institute Inc., 1988b).

For operational predictions of canopy air temperature on the basis of observations from ordinary meteorological stations only the CROPm and the CANOPY models are of interest. Unfortunately, these models did not fit the Formyre data too well, and neither did the models show acceptable prediction power in the 1988 and 1989 validations on Foulum data. For operational applications, a model having a prediction error above 1°C would probably not be of interest. As mentioned in chapter 5, the prediction error of the "best" model, i.e. the CROPm- R_C model, was 1.45 °C in 1988 and 1.79 °C in 1989.

Part of the original data set and some of the data manipulations applied in this study are open to criticism. Some important points should be mentioned: The $T_{c,z}$ observations in the Formyre data set appear to be subjected to radiation errors. Furthermore, the hourly values of $\Delta T_{c,z}$ used in the analyses were means of two replicates that occasionally differed substantially, possibly as a consequence of the radiation error mentioned above. Net radiation was measured above the barley canopy at Formyre, but above a lawn-type vegetation at Foulum (Fig. 5.10). The methods for calculating extinction coefficients for net radiation and wind speed within the canopy have not been validated. The assumption concerning the vertical CAI distribution has not been confirmed.

The models applied in this study are also open to criticism. The model hypothesis and consequently both the CROPm and the CANOPY models are most certainly far too simple formulations of the complicated bio-physical processes, that regulates the air temperature in the canopy. This point is demonstrated in particular by the results for the $\Delta T_{c,0.05}$ variable. The results show that the temperature regimes in the lower and the upper part of the canopy change differently in response to external factors. This behaviour might be expected as the temperature conditions in the lowermost part of the canopy are more strongly dependent on the thermal regime of the soil and on the internal energy transport processes in the canopy.

The 1988 and 1989 growing seasons were very much alike and did not differ much from a

"normal" growing season. The rather discouraging results in the 1988 and 1989 validation of the CROPm- R_C model indicate that the model would show an even worse performance when applied to "non-normal" growing seasons, stands of different plant density or barley fields situated remote to the meteorological station. In the latter case the topography of the neighbourhood becomes an important factor.

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A List of symbols

AIC Akaike Information Criterion aD_z, bD_z, cD_z, dD_z subsets of data set D_z , for different periods sensible heat flux, Wm^{-2} CCAI Crop Area Index CAI, CAI between heights z and z_c CANOPY statistical model in eq. (3.29)CROPa statistical model in eq. (3.28)CROPm-R statistical model in eq. (3.26, 3.27) $CROPm-\Omega$ statistical model in eq. (3.25)drag coefficient of leaves, c_d DAI "Dead-leaf" Area Index D(z')distribution function of CAI D_z Formyre data sets for height z D_z^+ subsets of D_z , only including observations with $Rn \ge 0 \text{ Wm}^{-2}$ $D_z^$ subsets of D_z , only including observations with $Rn < 0 \text{ Wm}^{-2}$ d zero plane displacement, m $\left[\frac{dRn_z}{dz}\right]^*$ average of $\frac{dRn_z}{dz}$ over 0.05 m vertical layer d(z')density function of CAI a function $f(\ldots)$ GSoil heat flux, Wm⁻² relative turbulence intensity, i_{ω} k von Karman's constant, 0.41 k_{Rn} extinction coefficient for net radiation in the canopy k_u extinction coefficient for wind speed in the canopy, -LAI Leaf Area Index l_m mean distance between leaves, m Ν number of observations \mathbb{R}^2 coefficient of determination of model RnNet radiation, Wm⁻² net radiation at height z in the canopy, Wm^{-2} Rn_z Rna net radiation term Sa storage term, Wm⁻² S_i global radiation, Wm⁻² s standard deviation $T_{c.z}$ canopy air temperature at height z above ground, $^{\circ}C$ $T_{2.0}$ air temperature at 2.0 m above ground, °C

u_c	horisontal wind speed at crop height z_c , ms ⁻¹
u_z	horisontal wind speed at height z , ms ⁻¹
u [′]	a wind speed term
u_*	friction velocity, ms ⁻¹
W	Shapiro-Wilk statistic
W_{obs}	observed Shapiro-Wilk statistic.
xAI'	a LAI, CAI or DAI term
z	height above ground level, m
z _c	crop height, m
z_c'	a crop height term
<i>z</i> ′	distance from top of canopy to the level z , m
z_0	roughness length, m
eta_1,eta_2,eta_3,eta_4	parameters in statistical models
$eta_1',eta_2',eta_3',eta_4'$	parameters in statistical models
$\Delta T_{c,z}$	difference $(T_{c,z} - T_{2,0})$ between air temperature in canopy at height z
	and temperature at height 2.0 m
λE	latent heat flux (evapotranspiration), Wm^{-2}
ε	statistical error, random variable
ρ	coefficient of correlation
$ ho_{obs}$	observed coefficient of correlation.
ϕ_1,ϕ_2	parameters in statistical models
Ω	sign conversion factor
ω	characteristic width of leaves, m
	autoregressive term in CROPa

B Derivation of some equations

B.1 Vertical distribution of crop area index, CAI

The density function of the vertical distribution of CAI was assumed to follow a "half-circle" function as shown in Fig. B.1. Below and in the text, the distribution is referred to as a "half-circle" distribution.



Figure B.1: Density function for the vertical distribution of CAI. The z, z' and x axes are introduced to facilitate calculations.

The vertical z, z' and x axes in Fig. B.1 are introduced to facilitate calculations of the CAI distribution and $\frac{dRn_z}{dz}$. The following relations between z, z' and x are easily verified

$$z' = -z + z_c \tag{B.1}$$

$$x = -z + \frac{1}{2}z_c \tag{B.2}$$

$$x = z' - \frac{1}{2}z_c \tag{B.3}$$

$$r = \frac{1}{2} \cdot z_c \tag{B.4}$$

The equation for the half-circel function may be written

$$f(x) = \sqrt{r^2 - x^2} \tag{B.5}$$

The integral of f(x), F(x), is given by

$$F(x) = \int f(x)dx = \frac{1}{2} \left[x\sqrt{r^2 - x^2} + r^2 \cdot \arcsin(\frac{x}{|r|}) \right]$$
(B.6)

Integration of f(x) over the interval [-r, x] yields

$$\int_{-\tau}^{x} f(x) \cdot dx = F(x) + \frac{\pi r^2}{4}$$
(B.7)

If we put x = r, (B.7) reads

$$\int_{-r}^{r} f(x) \cdot dx = \frac{\pi r^2}{2}$$
(B.8)

Combining (B.5) and (B.8) we find that the density function, d(x), of the "half-circle" distribution may be written

$$d(x) = \frac{2}{\pi r^2} \cdot \sqrt{r^2 - x^2}$$
(B.9)

and the corresponding distribution function, D(x), may be written

$$D(x) = \int_{-r}^{x} d(x)dx = \frac{2}{\pi r^2} [F(x) + \frac{\pi r^2}{4}]$$
(B.10)

B.2 Calculation of $\frac{dRn_z}{dz}$

The net radiation in the canopy Rn_z is calculated as

$$Rn_z = Rn \cdot \exp(-k_{Rn} \cdot CAI_{z'}) \tag{B.11}$$

where

$$CAI_{z'} = D(x) \cdot CAI = \frac{2}{\pi r^2} [F(x) + \frac{\pi r^2}{4}] CAI$$
 (B.12)

Differentiation of (B.11) with respect to $CAI_{z'}$ yields

$$\frac{dRn_z}{dCAI_{z'}} = -k_{Rn} \cdot Rn \exp(-k_{Rn}CAI_{z'})$$
(B.13)

Inserting (B.12) into (B.13) we find

$$\frac{dRn_z}{dx} = -d(x) \cdot CAI \cdot k_{Rn} \cdot Rn \exp(-k_{Rn} \cdot D(x) \cdot CAI)$$
(B.14)

observing from (B.2) that $\frac{dx}{dz} = -1$, (B.14) may be rearranged to read

$$\frac{dRn_z}{dz} = -\frac{dRn_z}{dx} = d(x) \cdot CAI \cdot k_{Rn} \cdot Rn \exp(-k_{Rn} \cdot D(x) \cdot CAI)$$
(B.15)

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Afdeling for Blomsterdyrkning, Kirstinebjergvej 10, 5792 Årslev	65 99 17 66
Afdeling for Frugt og Bær, Kirstinebjergvej 12, 5792 Årslev	65 99 17 66
Afdeling for Planteskoleplanter, Kirstinebjergvej 10, 5792 Årslev	65 99 17 66
Laboratoriet for Forædling og Formering, Kirstinebjergvej 10, 5792 Årslev	65 99 17 66
Laboratoriet for Gartneriteknik, Kirstinebjergvej 10, 5792 Årslev	65 99 17 66
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